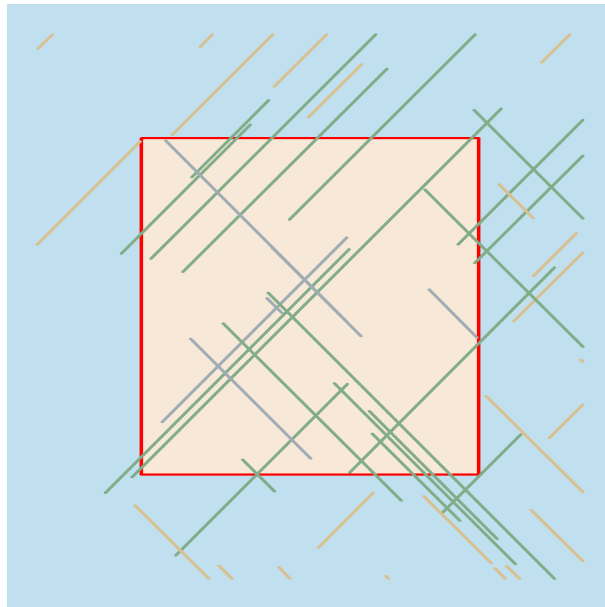


The Purpose of Computing

Leon Hannah Tabak



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For Ariel and Avidan

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Preface

0.1 What can be automated?

We want to know when a computer will be a useful tool, how to solve problems using a computer, how to evaluate the quality of our solutions, and how to establish confidence in the correctness of solutions. Computer science begins with efforts to answer these four questions.

1. For what kinds of questions can I write a program that will produce the answers?
2. Given a question for which I can write programs that will produce the answer, how do I write the program?
3. If we ask several people to write programs that solve a given problem and let them work independently, we should expect our programmers to deliver non-identical software. Given several programs that produce the answer for the same question, how do I select the best program?
4. If I am given a program, how can I be sure that the program solves exactly the problem we set out to solve.

0.2 What is a computer?

In the 1950s, soon after Univac, IBM, and other companies first began manufacturing computers, audiences saw computers in movies like *Desk Set* and *Forbidden Planet*. They recognized a computer when they saw an enormous cabinet decorated with blinking lights. The big machines buzzed, clattered, and beeped.

In the 1980s, soon after Apple, IBM, and other companies made personal computers practical, fictional heroes in movies like *WarGames* and *Star Trek IV: The Voyage Home* discovered the power of smaller computers. In the *Star Trek* story, the crew of the starship Enterprise traveled back in time to the 1980s. They landed in San Francisco. Scotty, the ship's chief engineer, needed a computer. He found an early model of the Apple Macintosh. He mistook the mouse for a microphone. Movie-goers laughed when they saw him try to talk to the computer.

We no longer see toggle switches and spinning reels of magnetic tape on the front faces of our computers. Thin liquid crystal displays have replaced hot, bulky cathode ray tubes. Twenty years ago, we copied software from floppy disks. Ten years ago, we purchased software on CD-ROMS. Now we fetch new software from the Internet. The form and appearance of our computers have changed. The physical features of our computers do not define them.

0.3 Impressive achievements

Computer scientists have created some of the most complex objects that human beings have ever designed and built. The largest programs contains tens of millions of instructions. If printed on paper and bound in books a single large program would fill hundreds of volumes. No single person can fully comprehend a large program. Large teams work for many years to create the most ambitious software. New members join and experienced contributors depart at every stage.

The smallness of electronic devices should impress us. The coordination of human beings in the development of software should impress us more.

Computer scientists seek ways of describing problems and methods for their solution concisely and unambiguously. Programmers must always write for at least two audiences. On the one hand, they write for a machine. On the other hand, they write for human teammates and clients. The contrast between the needs of human readers and the machines limitations challenges software engineers.

Sophisticated mathematics, clever codes, and inventiveness of individual programmers should impress us. Successful communication among diverse developers and their clients should impress us more.

0.4 Computability

For which problems do algorithms that generate a solution exist? This question defines the study of computability.

A programmer can direct a computer to repeat a series of arithmetic operations until some logical condition is satisfied. If the programmer fails to specify the terminating condition correctly, the computer will continue executing the instructions forever.

Will a given program complete its work in finite time?

Before anyone had built a computer, Alan Turing proved the impossibility of writing a program to answer that question. He supposed that a program-checking program could be written and then showed that a logical contradiction followed when he imagined using the program to check itself.

There are, of course, programs that check other programs for misspellings, unbalanced parentheses, and other kinds of errors. However, while it is possible to recognize an infinite loop in some programs, it is not possible to write a program that will reliably tell us whether or not any program that we give it contains an infinite loop.

0.5 Complexity

For computer scientists, the word complexity denotes the amount of work required to solve a problem using a given algorithm.

An algorithm is the sequence of arithmetic and logical operations that lead to the solution of a problem. Computer scientists measure the complexity of an algorithm by comparing the number of instructions that a computer executes in running the algorithm to completion with the size of the input given to the algorithm. If the complexity of a problem is very great, then a practical solution might remain out of reach even to the computer scientist who was an algorithm for the solution of the problem.

Here to illustrate the meaning of computational complexity are two problems:

- Checking to see if a persons name and number in a big citys telephone directory takes longer than finding a name and number in a small citys directory. How much longer?

- An eccentric hiker prefers to carry a knapsack that weighs exactly 42 pounds. At home, the hiker has a closet full of items that might be handy to have in the woods. The hiker has labeled each of those items with its weight. The time required to determine whether or not some combination adds up to the magic number 42 will depend upon the number of articles from which the hiker chooses. How does the time required to solve this problem relate to the number of items in the hikers closet?

You can find a name in a directory by repeatedly dividing the book in half.

- Hold the book in two hands. Open the directory at the middle with your thumbs.
- Directories list names alphabetically, so you can tell in which half to continue your search. Open to the middle of either the front or back half of the book.
- Divide that quarter into eighths, then the eighth into sixteenths, and so on until you have narrowed the search to a single page.
- If there is a listing, the page on which you land at the last step of your search will contain it.

The number of times we must divide a number by two until we reduce it to one is the logarithm base 2 of that number (rounded up). The function grows slowly. You will have to look at no more than 5 pages to find a name in a directory that contains 32 pages. You will have to look at no more than 6 pages in a 64 page directory, 10 pages in a 1000 page directory, 20 pages in a million page directory, and only 30 pages in a billion page book. Imagine even a billion billion trillion pageschecking 100 pages will get you to the page you want. The complexity of this algorithm (called binary search) is low.

You can answer the hikers question by computing the weight of every possible combination of items that the hiker might pack and then checking each sum against the 42 pounds that the hiker wants to carry. Although simple to describe, the algorithm has high complexity. The number of possible combinations (that is, the number of subsets of all items) is an exponential function of the number of items. Add one item to the hikers collection of gear and you double the number of subsets that you must consider.

(I will fib just a tiny bit in my next calculations because I want to establish a symmetry between this and the previous problem. If you are curious and clever, perhaps you can explain why you would write 31 where I have written 32.)

You must consider 32 combinations if you have 5 items, but 64 combinations if you have 6 items. If you are choosing items from a set of 10 items, you will

have to consider 1024 combinations. The number of combinations rises to a million if you have 20 items and to more than a billion if you have 30 items. If the hiker has stored 100 items in his closet, you will have to examine a billion billion trillion possible combinations.

You know that computers are fast. How fast? Can we compute a billion billion trillion sums in the time that God has given us on earth? Industry has increased the speed of computers at a rapid pace and promises continued improvements. How much gain in speed will we need to answer the hikers question? Suppose that a genie gave you such a powerful computer. What will you do if a second hiker presents you with 200 pieces of camping gear?

Many problems resemble the hikers problem in the following ways:

- We can describe the problems.
- We can describe algorithms for their solution.
- The times required to solve the problem with any of those known algorithms greatly exceed human lifespans, even on the fastest computers, except when the size of the input is very small.

Stephen Cook and Leonid Levin produced evidence that suggests that solutions to problems like the hikers problem are forever unattainable. They showed that an efficient solution to one problem in the set would allow an efficient solution to any of the others. Why should this knowledge strengthen our belief that each problem is intrinsically complex?

0.6 Notes on Searching and Sorting

When a word processing program checks the spelling in a document, the program searches for matching words in a dictionary. When a customer enters the title of a book on the Web site of an on-line bookstore, a program on the stores server searches for the price, reviews, and other information that will help the customer decide for or against the purchase of the book.

When the registrars office publishes the list of courses that the college offers, the office lists the courses in numerical order within departments that it lists alphabetically. Sorting is the key to efficient searches. Students can find the number of seats available in a course or the number of the classroom in which the class will meet quickly because the registrar has listed the courses in order.

An organization can save money by sorting newsletters by the zip codes of the members to which it wants to send the newsletters. The United States Postal Service charges less for postage when the organization sorts mail before bringing it to the post office.

We will study methods of searching and sorting because the solutions to many of the problems that clients present to software engineers require searching through data and/or sorting data. A large fraction of

Our examination of searching and sorting algorithms will lead us to a deeper understanding of challenges that are common to many kinds of programming problems. Of course, people who have never studied computer science know how to find the smallest element in a collection, find a matching value in a list, and order objects in set from smallest to largest. Searching and sorting are easy. However, specifying the steps taken in a search or sort is hard. Computer scientists can describe the procedures precisely.

Why program a computer to perform a task that people can do by hand? People can sort short lists by hand. Computers become indispensable when the number or length of the lists to be sorted become very large.

We study two algorithms for searching and three algorithms for sorting in our first course. If you continue your study of computer science, you will encounter other searching and sorting algorithms. By showing you several algorithms, we are making the point that you will often have a choice of methods for solving a given problem. A good computer scientist finds the solution to an assigned problem. A better computer scientist recognizes choices, selects the best alternative from among those several methods, and can give reasons for preferring one method over another.

0.6.1 Examples of sorting

		Best guess				
	5	2	3	1	4	
Step 1:	5	2	3	1	4	5
Step 2:	5	2	3	1	4	2
Step 3:	5	2	3	1	4	2
Step 4:	5	2	3	1	4	1
Step 5:	5	2	3	1	4	1
						1

Figure 1: Sequential search for minimum value.

		Best guess				
	5	2	3	1	4	
Step 1:	5	2	3	1	4	0
Step 2:	5	2	3	1	4	1
Step 3:	5	2	3	1	4	1
Step 4:	5	2	3	1	4	3
Step 5:	5	2	3	1	4	3
						3

Figure 2: Sequential search for index of minimum value.

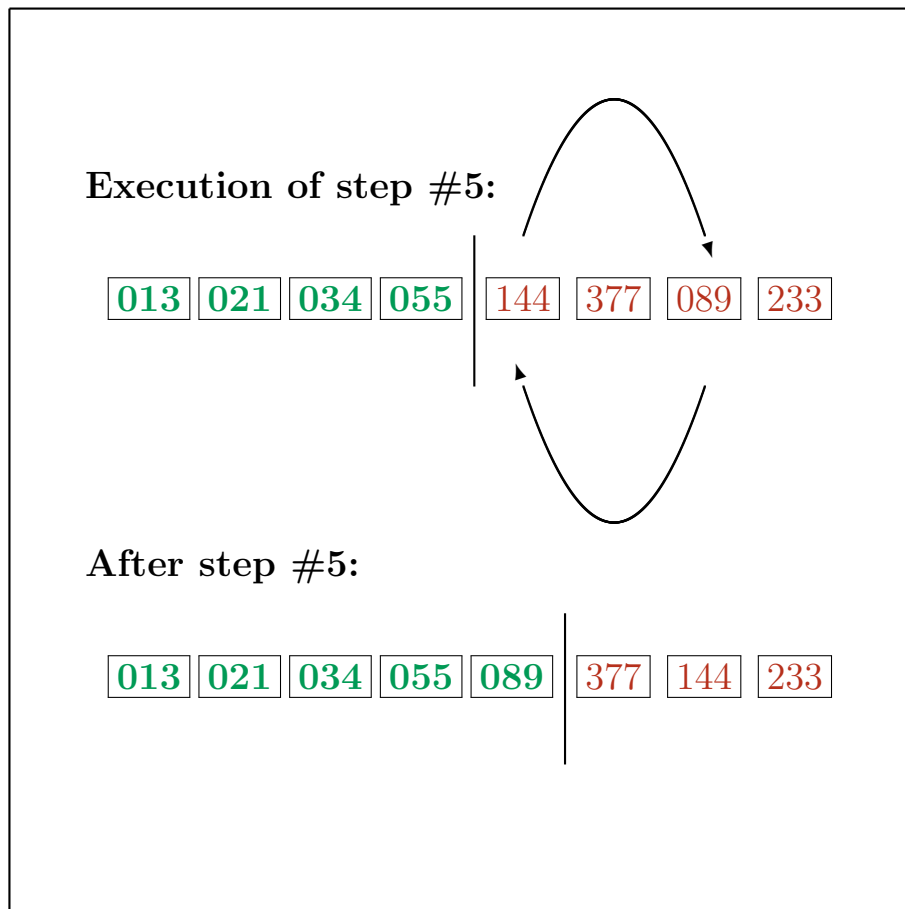


Figure 3: Selection sort.

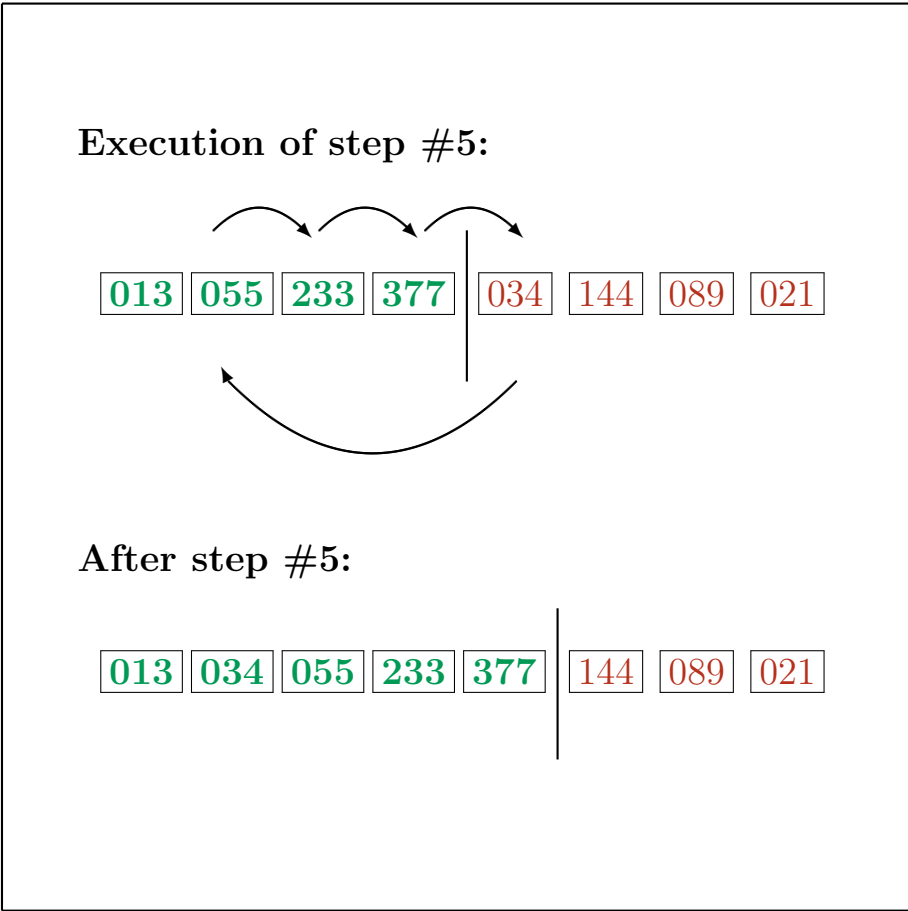


Figure 4: Insertion sort.

	<	><	34	48	83	36	75	47	36	24	>
Step 1	<	34	><	48	83	36	75	47	36	24	>
Step 2	<	34	48	><	83	36	75	47	36	24	>
Step 3	<	34	48	83	><	36	75	47	36	24	>
Step 4	<	34	36	48	83	><	75	47	36	24	>
Step 5	<	34	36	48	75	83	><	47	36	24	>
Step 6	<	34	36	47	48	75	83	><	36	24	>
Step 7	<	34	36	36	47	48	75	83	><	24	>
Step 8	<	24	34	36	36	47	48	75	83	><	>
	<	24	34	36	36	47	48	75	83	><	>

Figure 5: Insertion sort.

0.7 Lessons in “Hidden Figures”

The women whose stories are told in the movie “[Hidden Figures](#)” give us good examples of professional conduct. Some people have jobs and some people have a calling.

The attitudes and habits that distinguish the women as members of a profession might not be obvious to all viewers of the movie. Margot Lee Shetterly tells the story more fully than the movie could in her book ([“Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race”](#)).

Of course, Mary Jackson, Katherine Goble Johnson, and Dorothy Vaughan (the heroines in this story) are honest. However, ethical conduct in professional life means more than choosing not to lie, steal, and cheat.

Professional people take responsibility for their own learning. They continue learning throughout their lives. They help colleagues, especially junior colleagues, advance. They share their enthusiasm and their special knowledge and skills with their communities. They stand by their work. They defend their own work. They do not wait for directions, but exercise initiative. They anticipate problems. They make their product as good as it possibly can be.

	<	><	16	29	69	88	52	23	79	97	>
Step 1	<	16	><	29	69	88	52	23	79	97	>
Step 2	<	16	23	><	69	88	52	29	79	97	>
Step 3	<	16	23	29	><	88	52	69	79	97	>
Step 4	<	16	23	29	52	><	88	69	79	97	>
Step 5	<	16	23	29	52	69	><	88	79	97	>
Step 6	<	16	23	29	52	69	79	><	88	97	>
Step 7	<	16	23	29	52	69	79	88	><	97	>
Step 8	<	16	23	29	52	69	79	88	97	><	>
	<	16	23	29	52	69	79	88	97	><	>

Figure 6: Selection sort.

The book tells a story that begins during the Second World War. The women calculated the magnitudes of forces that affected airplanes in flight. Row upon row of human calculators entered numbers one at a time into **calculators** by hand. When jet engines replaced propellers and piston engines, they adapted. When the nation called upon their laboratory to help develop spacecraft, they added to their repertoire of mathematical methods. They foresaw the rapid development of computers and the end of the old ways of generating tables of numbers. The women organized their own courses. They taught themselves and one another. The human computers learned to use electronic computers.

Appreciative of the opportunities that they had found, they volunteered in churches and scout troops and other organizations. They spoke about the rewards that the engineering profession offers. They encouraged young people to invest themselves in work that offers great challenges and great rewards.

They asked “what if?” and developed a solution for what appeared to be an improbable mishap years before the same kind of accident crippled the Apollo 13 spacecraft.

Junior members of a team, when they choose to claim professional status, can defend their work confidently against challenges from higher ranking colleagues. The junior member is the expert in her own domain. In this story, a courageous defense not only demonstrates the integrity of one woman's work but also reveals

errors in the data that she had been given, and so improves the whole teams output.

The movies makers tell the story in compressed time. As the director pointed out in a [conversation with the Wall Street Journal](#), movie-goers are willing to watch a mathematician calculate for 30 seconds, but not for 3 days. Because they work in a visual medium, movie-makers prefer action. People run between buildings. Rockets hurtle through the sky.

On most days, most engineers do not run or fly. Engineers check and re-check, test, validate, and patiently make progress through the accumulation of many small improvements. Discipline, attention to detail, and perseverance also characterize professional conduct.

Read the [Software Engineering Code of Ethics](#) to learn more about how to be a responsible and professional person. Although written for software engineers, the principles are relevant in other professions too. What is your calling?

0.8 More reading

- [“On Computable Numbers, with an Application to the Entscheidungsproblem,”](#) by Alan Turing
- [“A Symbolic Analysis of Relay and Switching Circuits,”](#) by Claude Shannon
- [“I, Robot,”](#) by Isaac Asimov
- [“First Draft of a Report on the EDVAC,”](#) by John von Neumann
- [“As We May Think,”](#) by Vannevar Bush
- [“The Nine Billion Names of God,”](#) by Arthur C. Clarke
- [“The Human Use of Human Beings,”](#) by Norbert Wiener
- [“Algorithm 64: Quicksort,”](#) by C.A.R. Hoare
- [“A Note on Two Problems in Connexion with Graphs,”](#) by E.W. Dijkstra
- [“The Complexity of Theorem Proving Procedures,”](#) by Stephen Cook
- [“A Person Computer for Children of All Ages,”](#) by Alan Kay
- [“The GNU Manifesto,”](#) by Richard Stallman

Also, see [“The GNU Manifesto Turns Thirty”](#), by Maria Bustillos in *The New Yorker* magazine.

- “No Silver Bullet Essence and accidents of software engineering,” by Fred Brooks
- “The Cathedral and the Bazaar: Musings on Linux and Open Source by an Accidental Revolutionary,” by Eric Raymond

Also, listen to [Russ Robert’s interview with Eric Raymond](#) on the [Econ-Talk](#) podcast titled “Eric Raymond on Hacking, Open Source, and the Cathedral and the Bazaar.”

Lesson 1

The Java programming language

I will introduce you to the Java programming language today. We will discuss reasons for using this language rather than another in our course. We will compare programming languages to the languages that people speak.

1. Benjamin Whorf promoted the idea of linguistic relativity.
 - Who was he?
 - What is “linguistic relativity?”
2. Wilhelm von Humboldt also developed ideas related to linguistic relativity.

We will learn the Java programming language. Von Humboldt studied a language of the people of the island of Java.

Tell us something more about Wilhelm von Humboldt.
3. *Das Weltbild*, *die Weltanschauung*, and *die Weltsicht* are three German nouns with similar meanings. The meanings are related to the ideas developed by Whorf and von Humboldt. Look up the definition of one of these words. (If you are feeling more ambitious, try to find how their meanings differ.) Can you see the connection between the meanings of these words and the meaning of linguistic relativism?
4. What do the following German words mean?
 - (a) das Erklärungsnot
 - (b) das Fremdschämen

- (c) das Torschlusspanik
- (d) das Treppenwitz
- (e) verschlimmbessern

5. Think of how the German words in the previous question might find application in a translation of the following story:

You have found in the company that you and a partner have created a once in a lifetime opportunity. You have invested everything you have. For a year, you have been leading an effort to develop software on which the company has bet its future. You are racing to get to market before bigger companies that have more resources can do the same.

You are close to finishing an excellent design. Now you worry that more work might compromise the simplicity, soundness, and elegance of your design. You have promised delivery to your most important client by the end of the month.

Yesterday, your team met with the client. Your partner had insisted on presenting the product to this client, but bungled the demonstration and could not answer the client's questions. Your partner did not know enough to feel embarrassed. You squirmed. Then, without warning, your partner suddenly called you to the stage. You saved the day.

On the drive home, you thought of what you should have said to your ill-prepared partner after the nearly disastrous meeting.

-
- Do you think it is possible to produce a German version that conveys not just the same facts, but also the same emotions, assumptions, and values?
 - Does the language we use limit what we can say?
 - Are there ideas that we can express in one language but not another?
6. **The Imitation Game** is a motion picture that was released in 2014. It tells the story of Alan Turing.
Who was Alan Turing?
7. What is a Turing machine?
8. What is the Church-Turing Thesis?
9. Why should we use the Java programming language?
Search on the Web. What do other people have to say?

10. Learn a little about the origins, applications, and popularity of these other programming languages.

- C#
- JavaScript
- Python
- Ruby
- Scala

Lesson 2

2.1 A proof

I am going to prove to you that the number of people who have shaken hands with an odd number of people is even. This is true now. It is true of all of the people who have ever lived. It will always be true.

I will use a method called induction. Induction is a way of moving from specific facts to general propositions. Induction stands in contrast to deduction.

Deduction is a way of going from general knowledge to specific facts. Sherlock Holmes uses deduction. He possesses a great knowledge of history, science, literature, and other subjects. His general principles include, for example, laws of science. Drawing upon that general knowledge, he is able to deduce the identity of the particular person who committed a crime.

Induction begins with a few observations, typically about small cases. From those, we attempt to derive a rule that applies to all cases.

For the handshake problem, let us go back in time.

There was a time before anyone had shaken hands with anyone. Zero people had shaken hands with an odd number of people. Zero is an even number. Before the invention of the handshake, our proposition was true: the number of people who had shaken hands with an odd number of people was even.

There must have been a first handshake. Let us suppose that Adam, who was a gentleman, upon meeting Eve, greeted her and extended his hand. In this way, Adam and Eve invented the handshake. Two people each shook hands with one other person. Two is an even number. One is an odd number. After Adam and Eve concluded the first handshake, an even number of people had shaken

hands with an odd number of people. This is a second instance in which our proposition was true.

Now I will show that if our proposition was ever true, it must be forever after true.

There are only two kinds of people in the world. We will call the first kind EVEN. These are the people who have shaken hands with 0, 2, 4, 6, or some other even number of people. We will call the second kind ODD. These are the people who have shaken hands with 1, 3, 5, or some other odd number of people.

There are only three kinds of handshakes.

CASE 1: An EVEN person shakes hands with another EVEN person. Since an even number plus one is an odd number, with this handshake two people leave the EVEN group and join the ODD group. This handshake increases the size of the ODD group by two. If the number of people in the odd group was even before the handshake, it is also even after the handshake. (An even number plus two is another even number.)

CASE 2: An ODD person shakes hands with another ODD person. Since an odd number plus one is an even number, with this handshake two people leave the ODD group and join the EVEN group. This handshake decreases the size of the ODD group by two. If the number of people in the odd group was even before the handshake, it is also even after the handshake. (An even number minus two is another even number.)

CASE 3: An EVEN person shakes hands with an ODD person. With this handshake, the EVEN person joins the ODD group (an even number of handshakes plus one more handshake is an odd number of handshakes) and the ODD person leaves the ODD group to join the EVEN group (an odd number of handshakes plus one more handshake is an even number of handshakes). One person leaves the ODD group and the other person joins the ODD group. The handshake leaves the number of people in the ODD group unchanged. If the number of people in the ODD group was even before, it is still even.

Every handshake either (1) adds two to the size of the ODD group, (2) subtracts two from the size of the ODD group, or (3) adds zero to the size of the ODD group. If the number of people in the ODD group was ever even, it must forever after remain even. We know that the number was once even (remember Adam and Eve). Our reasoning leads to the conclusion that the number is still even and that it will always be even.

This is a theorem in a branch of mathematics called graph theory. In this context, a “graph” is a network of nodes and connecting edges. A map of the routes that an airline flies is a graph and so is a diagram of a computer network. In those kinds of maps and diagrams, scale is not important but information about which nodes are connected (for example, which cities are linked by direct flights) is important. UPS uses graphs to plan routes for its deliveries. A knowledge of the properties of graphs such as the one that we have just proved can help in the solution of that and other practical problems.

2.2 A way of testing software

“Test-driven development” means writing tests first and the software that must pass the tests second. I introduce students to this discipline for creating software in my classes. We use JUnit, a popular collection of Java classes (components for our programs that have already been written for us) together with an integrated development environment like NetBeans or Eclipse.

Typically, I begin by showing students a program that models arithmetic with fractions. This program requires a class (a blueprint for the construction of individual fractions) that models a fraction. The class specifies a numerator and a denominator plus methods for adding, subtracting, and so on. In the code that I give to the students at the start of the exercise, the addition method produces a sum of 0/1 for every two fractions that are given to it. This is a “stub” function—it has the correct form but incomplete (and initially incorrect) functionality. Correct form here means that we specify that we need two fractions and we expect to get back a third fraction.

(The author of a stub method defers decisions about how to produce a correct result. Because the engineer must provide some result in order to give the method the right form, the engineer specifies an arbitrary result like zero. Then every engineer on the development team can compile and run the program. They can continue work on other parts of the program, then return to the incomplete stub method at a later time.)

My tools (the NetBeans/JUnit combination) looks at that nearly empty code and automatically generates most of the code that tests it. I have to edit the generated code only a little to say which two fractions I want to add and to give the sum that I know is correct. In the same fashion, I specify correct results for examples of subtraction, multiplication, and division of fractions in the generated code that will execute my tests.

The code that I give to my students compiles and runs. The students see a window in which the name of each method (add, subtract, and so on) appears and

with each name a red "X" that signals the incorrectness of that method's definition. As they complete the definitions of each method, the software replaces each red "X" with a green check mark.

The word "unit" in the name "JUnit" identifies this tool as a tool for unit testing. "Unit testing" is testing at the lowest level. In a unit test, software engineers test the smallest elements of the product. These smallest elements are the individual classes and their methods. Unit testing stands in contrast to modular testing, system testing, and integration testing.

Unit testing is also linked to regression testing. "Regression testing" means re-testing. It often happens that the bugs we fixed last month reappear this month. Software is almost always the product of a team. If one member of a team misread a specification or made an incorrect assumption, another member of the team might do the same. Therefore, it is not enough to fix a bug, run the test, and get all green check marks. We have to keep running the tests forever. Tools like JUnit have made it much easier to repeatedly execute large numbers of tests because they have very largely automated their construction and execution.

When I and several colleagues and students visited SkyWorks (a company that makes chips for cell phones), the engineers explained that one of their suites of tests that had included 100 tests the year before now includes 10,000 tests.

Many organizations that have adopted the agile approach for the development of products run large suites of tests every single day. They test the whole product. They ensure that they have a working product at the end of every day. In older, non-agile approaches, members of a team sometimes worked weeks or months apart before bringing their separate contributions together. The code between release versions was therefore sometimes unusable. Agile development calls for rapid iteration—frequent, small improvements that are shared with the whole team and the clients. The agile approach integrates testing (quality assurance) throughout the product's life cycle. Testing is not saved until the end!

When we write unit tests, we are writing a specification of our product. In this way, we are contributing to the design of the product. Even when we commit to test-driven development, we expect to compose new tests in every phase of a product's life. The test suite is, like specifications, design diagrams, and instructions for the customers/clients, part of the product.

Finally, a familiarity with white box and black box testing might help you in an interview.

In white box testing, the engineer who writes the tests can examine the inner workings of the product that is to be testing. This has the advantage of enabling the engineer the opportunity to test every branch of every "if" statement. It

allows more comprehensive testing.

In black box testing, the test engineer knows what the product is supposed to do but does not know how the design engineers chose to create that particular functionality. This has the advantage of hiding the design engineers' biases from the test engineer's eyes. It reduces the risk that the test engineer will make the same false assumptions as did the design engineers.

The ideas and vocabulary that I have shared with you here have application outside of software engineering.

Lesson 3

3.1 SequentialSearch.java

```
package aufgaben;

import java.util.Random;

public class SequentialSearch {

    private final static Random rng = new Random();

    private static int[] makeArray(int size, int range) {
        int[] result = new int[size];
        for (int i = 0; i < result.length; i++) {
            result[i] = rng.nextInt(range);
        } // for
        return result;
    } // makeArray( int )

    private static void printArray(int[] data) {
        for (int n : data) {
            System.out.print(n + " ");
        } // for
        System.out.println();
    } // printArray( int [] )

    private static boolean isContainedInArray( int n, int [] data ) {
        boolean result = false;
        for( int i = 0; i < data.length; i++ ) {
            if( n == data[i] ) {
                result = true;
            } // if
        }
    }
}
```

```
        } // for
        return result;
    } // isContainedInArray( int, int [] )

    public static void main(String[] args) {
        int[] list = makeArray(12, 24);
        printArray(list);
        System.out.print( "Is 17 in in the array? " );
        System.out.println( isContainedInArray( 17, list ) );
    } // main( String [] )

} // SequentialSearch
```

3.2 FindMinimum.java

```
package aufgaben;

import java.util.Random;

public class FindMinimum {

    private final static Random rng = new Random();

    private static int [] makeArray(int size, int range) {
        int [] result = new int [size];
        for (int i = 0; i < result.length; i++) {
            result [i] = rng.nextInt (range);
        } // for
        return result;
    } // makeArray( int )

    private static void printArray(int [] data) {
        for (int n : data) {
            System.out.print (n + " ");
        } // for
        System.out.println ();
    } // printArray( int [] )

    private static int smallestValue(int [] data) {
        int bestGuessSoFar = data [0];
        for (int i = 1; i < data.length; i++) {
            if (data [i] < bestGuessSoFar) {
                bestGuessSoFar = data [i];
            } // if
        } // for
        return bestGuessSoFar;
    } // smallestValue( int [] )

    public static void main(String [] args) {
        int [] list = makeArray (12, 24);
        printArray (list);
        System.out.println ( "Smallest value in array = " +
            smallestValue ( list ) );
    } // main( String [] )
} // FindMinimum
```

3.3 FindPositionOfMinimum.java

```
package aufgaben;

import java.util.Random;

public class FindPositionOfMinimum {
    private final static Random rng = new Random();

    private static int[] makeArray(int size, int range) {
        int[] result = new int[size];
        for (int i = 0; i < result.length; i++) {
            result[i] = rng.nextInt(range);
        } // for
        return result;
    } // makeArray( int )

    private static void printArray(int[] data) {
        for (int n : data) {
            System.out.print(n + " ");
        } // for
        System.out.println();
    } // printArray( int [] )

    private static int positionOfSmallestValue(int[] data) {
        int bestGuessSoFar = 0;
        for (int i = 1; i < data.length; i++) {
            if (data[i] < data[bestGuessSoFar]) {
                bestGuessSoFar = i;
            } // if
        } // for
        return bestGuessSoFar;
    } // positionOfSmallestValue( int [] )

    public static void main(String[] args) {
        int[] list = makeArray(12, 24);
        printArray(list);
        System.out.println("Position of smallest value in array = " +
            positionOfSmallestValue(list));
    } // main( String [] )
}
```

3.4 FindPosOfMinStartingAtIndex.java

```
package aufgaben;

import java.util.Random;

public class FindPosOfMinStartingAtIndex {

    private final static Random rng = new Random();

    private static int [] makeArray(int size, int range) {
        int [] result = new int [size];
        for (int i = 0; i < result.length; i++) {
            result [i] = rng.nextInt (range);
        } // for
        return result;
    } // makeArray( int )

    private static void printArray(int [] data) {
        for (int n : data) {
            System.out.print (n + " ");
        } // for
        System.out.println ();
    } // printArray( int [] )

    private static int positionOfSmallestValue(int [] data, int start ) {
        int bestGuessSoFar = start;
        for (int i = start + 1; i < data.length; i++) {
            if (data [i] < data [bestGuessSoFar]) {
                bestGuessSoFar = i;
            } // if
        } // for
        return bestGuessSoFar;
    } // positionOfSmallestValue( int [] )

    public static void main(String [] args) {
        int [] list = makeArray(12, 24);
        printArray (list);
        System.out.println ("Position of smallest value in array = "
            + positionOfSmallestValue (list, 6));
    } // main( String [] )
} // FindPosOfMinStartingAtIndex
```

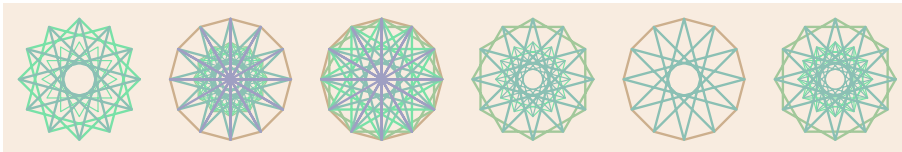

Lesson 4

“As I have always held it a crime to anticipate evils, I will believe it a good comfortable road until I am compelled to believe differently.”

—Meriwether Lewis [1]

“All programmers are optimists. Perhaps this modern sorcery especially attracts those who believe in happy endings and fairy godmothers. Perhaps the hundreds of nitty frustrations drive away all but those who habitually focus on the end goal.”

—Frederick P. Brooks [2, page 35]



4.1 Video clips

Here are links to trailers for several movies in which computers have a leading role.

- “Forbidden Planet” (1956)

- “Desk Set” (1957)
- “2001: A Space Odyssey” (1968)
- “Bicentennial Man” (1999)
- “Her” (2013)

As you view each of these clips, think of how a viewer at the time of the film’s release might have answered these questions:

- If we are told that there is a computer in the room that we have just entered, how will we recognize the computer?
- What are the essential features of a computer?
- Do we expect greater promise or greater peril with advances in computing technology?
- How will computers change our lives?

4.2 Binary numbers

Engineers have found it convenient to build computers that represent integers in binary (base 2) form. This is not an essential feature of computers. It is possible to build computers that work with decimal numbers. It is just that binary arithmetic makes designing digital electronics easier.

Different levels of voltage represent different digits. In designing a computer whose native language is binary numbers, engineers only have to distinguish between two levels of voltage. That makes the engineers’ job easier. It allows simpler circuits.

You know that...

$$\begin{aligned}1905 &= 1 \cdot 10^3 + 9 \cdot 10^2 + 0 \cdot 10^1 + 5 \cdot 10^0 \\ &= 1000 + 900 + 0 + 5\end{aligned}$$

This is place value arithmetic. In decimal numbers, each digit represents a multiple of a power of ten. In binary numbers, each digit represents a multiple of a power of two.

$$\begin{aligned}
 101010_2 &= 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 \\
 &= 32 + 8 + 2 \\
 &= 42_{10}
 \end{aligned}$$

To compute the product of two powers of the same number, add the exponents:

$$\begin{aligned}
 2^2 &= 2 \cdot 2 \\
 &= 4 \\
 2^3 &= 2 \cdot 2 \cdot 2 \\
 &= 8 \\
 2^5 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\
 &= (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2) \\
 &= 2^3 \cdot 2^2 \\
 &= 8 \cdot 4 \\
 &= 32 \\
 2^5 &= 2^3 \cdot 2^2 = 2^{3+2}
 \end{aligned}$$

Every computer scientist should know some powers of two by heart and be able to produce others. The table in Figure 4.1 will help you get started.

Commit $2^8 = 256$ and $2^{10} = 1024$ to memory. Learn how to calculate and estimate the values of other powers of two. Figure 4.1 includes models that show you how to do this. For example, $2^{12} = 2^{10} \cdot 2^2 = 1024 \cdot 4 \approx 1000 \cdot 4$.

4.3 Exercises

1. Look at the keyboard on your computer. Estimate the total number of different letters (count uppercase and lowercase!), digits, punctuation marks. Round up to the nearest power of two. The exponent on that power of two is the minimum number of bits required to specify a character.

If we want to construct a code that assigns a binary number to each character that we can type, how big (how many bits) must that number be?

2. How many meters are in a kilometer? How many bytes are in a kilobyte? Be careful!

$$\begin{aligned}2^0 &= 1 \\2^1 &= 2 \\2^2 &= 4 \\2^3 &= 8 \\2^4 &= 2^2 \cdot 2^2 = 4 \cdot 4 = 16 \\2^5 &= 32 \\2^6 &= 2^4 \cdot 2^2 = 16 \cdot 4 = 64 \\2^7 &= 128 \\2^8 &= 2^4 \cdot 2^4 = 16 \cdot 16 = 256 \\2^9 &= 512 \\2^{10} &= 1024 \\2^{11} &= 2048 \\2^{12} &= 2^{10} \cdot 2^2 = 1024 \cdot 4 = 4096 \\2^{13} &= 8192 \\2^{14} &= 16,384 \\2^{15} &= 32,768 \\2^{16} &= 2^{10} \cdot 2^6 = 1024 \cdot 64 = 65,536 \\&\vdots \\2^{20} &= 2^{10} \cdot 2^{10} = 1024 \cdot 1024 = 1,048,576 \\&\vdots \\2^{32} &= 4,294,967,296 \\&\vdots \\2^{64} &= 18,446,744,073,709,551,616\end{aligned}$$

Figure 4.1: Powers of 2.

3. 2^{10} is approximately one thousand. Therefore, $2^{20} = 2^{10} \cdot 2^{10}$ is approximately equal to what number?
4. Which powers of two correspond to the prefixes “mega,” “giga,” and “tera?” Approximately which powers of ten correspond to the same prefixes?
5. The Java platform includes a class named Color. That class defines a constructor that has three integer parameters—to create a color, the programmer specifies the amount of red, green, and blue in the color with three integers. The values must lie within the range 0 to 255—there are 2^8 different possible values of red, 2^8 possible values of green, and 2^8 possible values of blue. The total number of possible colors is then $2^8 \cdot 2^8 \cdot 2^8$.
Approximately how many millions of different colors are possible?
6. The arithmetic that you see in a course in computer science will sometimes differ from the arithmetic you see in courses on mathematics!

Copy this Java source code into a file on your computer. Compile and execute it.

Which results surprise you?

```

package arithmetic;

public class Arithmetic {
    private static final int BILLION = 1000000000;

    public static void main( String [] arguments ) {
        String label = "1 billion + 1 billion = ";
        int sum = BILLION + BILLION;
        System.out.println( label + sum );

        label= "1 billion + 1 billion + 1 billion = ";
        sum = BILLION + BILLION + BILLION;
        System.out.println( label + sum );

        double x = 8.0/0.0;
        String s = "(8.0/0.0)";
        System.out.println( s + " = " + x);
        System.out.println( "-" + s + " = " + -x);
        System.out.println( s + " + " + s + " = " + (x + x));
        System.out.println( s + " - " + s + " = " + (x - x));
        System.out.println( s + " * " + s + " = " + (x * x));
        System.out.println( s + " / " + s + " = " + (x / x));

        System.out.println( "5 / 3 = " + (5/3));
        System.out.println( "5.0 / 3.0 = " + (5.0/3.0));
    }
}

```

```
System.out.println( "5 % 3 = " + (5%3));
System.out.println( "3 * (5/3) + (5%3) = " +
                    (3*(5/3) + (5%3)));

System.out.println( "3.0 * (5.0/3.0) + (5.0%3.0) = " +
                    (3.0*(5.0/3.0) + (5.0%3.0)));

    } // main( String [] )
} // Arithmetic
```

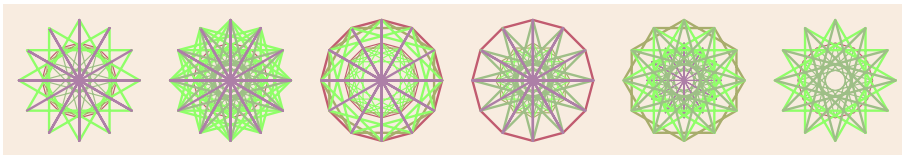
Lesson 5

“I think plans are highly overrated. Plans are a baseline, in my opinion, a model of a life that you depart from as you go on. Very rarely do they take you to the place you envisioned at the time you started.”

—Andrew S. Grove [10]

“I say ‘innovation’ rather than simply ‘invention,’ because innovation, to me, means invention implemented. And I have grudgingly come to realize that invention is often the easy part of innovation. The hard part is usually the implementation.”

—John Seely Brown [5]



5.1 Video clips

Here are links to several advertisements for the Intel Corporation.

- “Big Ideas”

- “Our Co-workers”
- “Our Doodles”
- “Our Parties”
- “Rock Star”

- | |
|--|
| <ul style="list-style-type: none">• Why are these advertisements funny?• What are some of the characteristics commonly associated with engineers?• Which of the attributes would you like to cultivate in yourself?• What does the word “intel” suggest? What is the origin of the name of the Intel corporation? |
|--|

5.2 A young field?

Computer scientists continue to regard their field as a young field. For how much longer can we call computer science young? To what else are people comparing computer science when they call it young?

Progress has been very rapid in some respects. Moore’s Law may be the best known example and description of fast-paced developments in computing. (You do not know Moore’s Law? Look it up!) At the same time, it has sometimes taken decades for important ideas to move from laboratories to widespread practice in industry. RISC microprocessors, object-oriented programming, and support for functional programming in the most widely used programming languages are cases in point.

Some perspective will help us understand how innovation proceeds. With an understanding of the past, we can form realistic expectations of the future.

Computer science has produced a lot of jargon.

5.3 Exercises

1. Construct a timeline of the history of computing. Associate with each point the name of one of the people listed in Figure 5.1 together with an achievement of that person. An achievement could be the publication of an influential article or book, an invention or discovery, the creation of

Hal Abelson	Marc Andreessen	Isaac Asimov
John Atanasoff	Charles Babbage	John Backus
Steve Balmer	Gordon Bell	Leo Beranek
Fred Brooks, Jr.	Vannevar Bush	Vint Cerf
Edgar F. (Ted) Codd	Stephen Cook	Seymour Cray
Peter J. Denning	Edsger Dijkstra	John Doerr
J. Presper Eckert	Richard Feynman	Carly Fiorina
Bill Gates	Adele Goldberg	Herman Goldstine
Shafi Goldwasser	James Gosling	Andy Grove
Richard Hamming	John Hennessy	Danny Hillis
C.A.R. (Tony) Hoare	Herman Hollerith	Grace Murray Hopper
Steve Jobs	Bob Kahn	Richard Karp
Gary Kasparov	John Kemeny	Vinod Khosla
Jack Kilby	Donald E. Knuth	Ray Kurzweil
John Lasseter	Gottfried Wilhelm Leibniz	J.C.R. Licklider
Barbara Liskov	Ada Lovelace	John Mauchly
Marissa Mayer	John McCarthy	Marvin Minsky
Gordon Moore	Peter Naur	Robert Noyce
Ken Olsen	David Patterson	Radia Perlman
Eric Raymond	Ginni Rometty	Ken Sakamura
Claude Shannon	William Shockley	Balaji Srinivasan
Richard Stallman	Cliff Stoll	Bjarne Stroustrup
Ivan Sutherland	Andy Tanenbaum	Linus Torvalds
Alan Turing	John von Neumann	Thomas Watson, Jr.
Meg Whitman	Norbert Wiener	Maurice Wilkes
Jeanette Wing	Niklaus Wirth	Steve Wozniak
Konrad Zuse		

Figure 5.1: People

a company or product, an award, a prediction, or an appointment to an important office.

You do not each have to refer to every person in this list on your timeline—let's divide the work among everyone in the class.

Find the information that you need on the Internet.

2. Define the terms listed in Figure 5.2. Short statements will suffice in some cases. In other cases, you might find a story about the origin of the term to add to the definition.

You do not each have to define every term in this list—let's divide the work among everyone in the class.

Find the information that you need on the Internet.

ALGOL	ALU	ASCII	BASIC	bit
byte	CAD	CAE	CFG	CISC
CMOS	COBOL	CPU	CP/M	CSS
DBMS	DEC	DOS	ECL	ENIAC
FTP	GaAs	GNU	GPL	GUI
HAL 9000	HDL	HTML	IBM	IC
ICAAN	JAR	JPEG	JSON	LAMP
Lisp	MANIAC	MIME	MIPS	MP3
NFS	NP	OOP	PCB	PDP-11
PKE	PL/I	PNG	RAID	RAM
RISC	ROM	RSA	SMTP	SPARC
SUN	SQL	TCP/IP	TLD	TTL
UNIVAC	URL	VAX	VLSI	VME
VMS	WNT	WYSIWYG	XM	

Figure 5.2: Acronyms, initialisms, invented words, and portmanteaus.

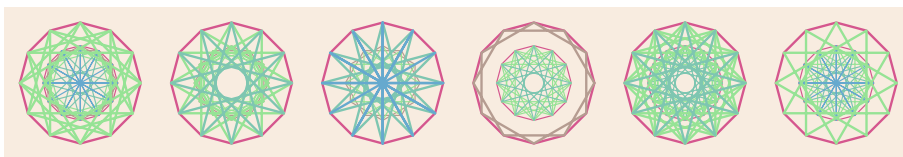
Lesson 6

“It’s the way I study—to understand something by trying to work it out or, in other words, to understand something by creating it. Not creating it one hundred percent, of course; but taking a hint as to which direction to go but not remembering the details. These you work out for yourself.”

—Richard P. Feynman [7, page 15]

“It is not a question of creativity versus discipline; creative work is simply not possible without discipline.”

—Watts S. Humphrey [12, page 26]



6.1 Review

1. Create a Web page that displays the words “Heuristically programmed ALgorithmic computer” in a top level headline. Use just the *DOCTYPE* declaration and the *body*, *h1*, *head*, *html*, and *title* tags.

2. Beginning in the 1940s, this prolific author established his reputation by writing science fiction. In many stories, he explored dilemmas that arise when intelligent robots try to obey the three laws that he invented.

The three laws are:

- A robot may not injure a human being or, through inaction, allow a human being to come to harm.
- A robot must obey the orders given to it by human beings, except where such orders would conflict with the First Law.
- A robot must protect its own existence as long as such protection does not conflict with the First or Second Laws.

Who is he?

3. He shared what his experience leading a large programming project for IBM in the early 1960s had taught him in *The Mythical Man-Month*. He later joked that the book had become “The Bible of Software Engineering” because, like the Holy Book, “everybody quotes it, some people read it, and a few people go by it.”

Who is he?

4. She studied computer science at Northwestern University. After a short stint with General Motors Institute, she joined IBM. She contributed in engineering, consulting, sales, and marketing roles. She led the effort to find commercial applications for IBM’s *Jeopardy* playing computer, Watson. Last year, she became IBM’s president and chief executive officer.

Who is she?

5. A professor emeritus at Stanford University, he pioneered methods of analyzing algorithms, wrote the enormously influential *The Art of Computer Programming*, and created software to typeset the book. Scientists and mathematicians around the world have subsequently adopted that software (called T_EX). They use it to write dissertations, articles, and books. He pays readers who find errors in his books, but no one cashes the checks.

Who is he?

6. While employed at the Digital Equipment Corporation, she invented a network protocol that greatly improved Ethernet, the very widely used local area network technology. She also invented an important algorithm that is used to route packets through the Internet.

You can find references to her as the “Mother of the Internet.” However, please do not call her that if you meet her. She does not like that accolade.

Who is she?

7. He studied physics at Cornell College, earned a doctoral degree at Harvard University, taught at the Massachusetts Institute of Technology, and created a company that built the first packet switches for the Internet.

His journey from Cornell College to Harvard University began when he offered to change the tire of a stranger who was passing through Mount Vernon.

Who is he?

8. A journalist, he once wrote for *Rolling Stone*, but then developed a fascination with the fast developing personal computers that he used for his writing. He has written books about the hacker ethic, Apple's creation of the Macintosh, Google, and cryptography.

Who is he?

6.2 Programming languages

Every programming language is like every other programming language in this respect: if it is possible to write a program in language A, it is possible to write a program in language B that produces exactly the same outputs when given the same inputs.

Every computer is like every other computer in this respect: it is possible to write a program on computer B that perfectly simulates the functions of computer A. Again, "perfect simulation" means that the same sequence of words and numbers fed into the two computers will cause the two computers to emit the same output (which might be signals, images, text, sounds, movements of a robot's arm, or something else).

Of course, there are also many important differences that distinguish the many programming languages and models of computers. We will give special attention to features of a language that make it easier for human beings to read and write programs.

Here is an outline of elements of the Java programming language:

Not all of the features listed here appear in all other languages because not all features are independent of one another. For example, if a programming language includes a means of adding positive and negative numbers, it does not need a separate operator for subtraction—a subtraction is just the addition of a negative number.

- data types

- primitive types: **boolean** (**true** or **false**), **double** (numbers that can have a fractional part), **int** (whole numbers) — and others that we will mostly ignore
- arrays (refer to individual elements with an index—for example, student[12])
- reference types—pointers to instances of classes (e.g., Color, PriorityQueue, Rectangle2D) (refer to individual elements with a period and the name of the field—for example, student.firstName)
- declarations—instruct the computer to allocate space in its memory for the storage of a value, give that location a name, and specify the type of value that will be stored
- expressions
 - operands
 - * literals (numbers, the words **true** and **false**)
 - * variables (names of locations in computer’s memories that hold values)
 - * calls to functions
 - operators
 - * binary arithmetic operators: +, -, % (remainder), *, /
 - * unary arithmetic operator(s): -
 - * binary logical operators: && (“and”), || (“or”)
 - * unary logical operator(s): ! (“not”)
- statements
 - assignment—instruct the computer to evaluate an expression and then store the expression’s value in a named location in the computer’s memory
 - **if**—instruct the computer to conditionally execute a statement
 - (**switch**)
 - **for**—instruct the computer to execute a statement repeatedly (a specified number of times)
 - **while**—instruct the computer to execute a statement repeatedly (so long as some condition still holds true)
 - (**do while**)
- divide a big program into units of manageable size
 - methods—a sequence of statements that has a single, well-defined purpose
 - classes—a blueprint for the construction of objects

- objects—a collection of related data and the methods for operating on that data (where “operating” might mean changing the data, retrieving that data, or combining that data with other data)
- compose small units—treat the assembly as a single unit
 - compound statement—an assembly of statements within a method
 - methods—an assembly of statements
 - class—an assembly of variables and methods
 - package—an assembly of related classes

6.3 Exercises

1. The name of which one of Java’s primitive types honors an English mathematician of the nineteenth century?
2. Java makes a distinction between whole numbers and numbers that may contain a fractional part (for example, 3.14159265) or an exponent (for example, Avogadro’s number $6.022 \cdot 10^{23}$ and the Planck constant $6.62 \cdot 10^{-34}$).

Which of the keywords of the Java programming language do programmers use to label variables to which they intend to assign values that might include a fractional part or an exponent?

3. Order the words “**class**,” “**method**,” and “**package**” so that the first item in your list is a something that can contain the second thing named in your list, and the second item is something that can contain the third thing named in your list.
4. Which of these delimiters do Java programmers use to mark the beginning and end of a sequence of statements: braces {}, parentheses (), or brackets [] ?

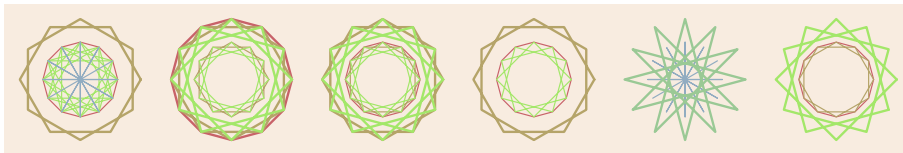
Lesson 7

“If we perceive our role aright, we then see more clearly the proper criterion for success: a toolmaker succeeds as, and only as, the *users* of his tool succeed with his aid. However shining the blade, however jeweled the hilt, however perfect the heft, a sword is tested only by cutting. That swordsmith is successful whose clients die of old age.”

—Frederick P. Brooks [4, page 62]

“The real significance of computing was to be found not in this gadget or that gadget, but in how the technology was woven into the fabric of human life—how computers could change the way people thought, the way they created, the way they communicated, the way they worked together, the way they organized themselves, even the way they apportioned power and responsibility.”

—M. Mitchell Waldrop [20, page 342]



7.1 Classes that have constructors.

One kind of a class is a blueprint the we use to construct objects. Each object is an “instance of the class.”

For example, Java provides a `Color` class. This class has a constructor that requires three integer arguments. The integers specify the amount of red, green, and blue in a color. A programmer can create any number of colors with the `Color`.

```
// Colors of the flag of the United States of America.
Color red = new Color( 255, 0, 0 );
Color white = new Color( 255, 255, 255 );
Color blue = new Color( 0, 0, 255 );
```

The `Color` class has many methods. These methods includes accessor methods that enable a programmer to retrieve the values of a color's red, green, and blue fields.

```
Color peach = new Color( 248, 206, 160 );
int howMuchRed = peach.getRed();
```

Every Java application has at least one class. It may contain many classes. At least one class in an application must contain a `main()` method.

A `main()` method looks like this:

```
public static void main( String [] args ) {
    // Statements go here.
} // main( String [] )
```

Although there is a way to define classes that do not need constructors, the classes that we will write will have constructors. (A class may have more than one constructor if the constructors have different numbers and types of parameters.)

If the name of a class is `MyClass`, then its constructor might look like this:

```
public MyClass() {
    // Statements go here.
} // MyClass()
```

We will begin our definition of classes by writing something like this:

```
public class MyClass {
    public MyClass() {
        // Statements go here.
    } // MyClass()

    // Other methods go here.

    public static void main( String [] args ) {
```



```

    MyClass myClass = new MyClass ();
} // main( String [] )
} // MyClass

```

7.2 Class that have no constructors.

The Math differs from the Color class in this important way: we might want to create many colors (each with different amounts of red, green, and blue) but we have no need to create multiple Math objects. Mathematics is the same everywhere and always.

The Math class is not a blueprint. It is a bundle of related methods (square root, sine, cosine, logarithm, and others) and constants (the numbers pi and e). All of the methods in the Math class have the **static** attribute (just like the main()) The **static** qualifier makes these methods callable even before we create an instance of a class.

To call a **static** method, write the name of the class that contains the method, a period, and then the name of the method:

```
double logarithm = Math.log( 1.0 );
```

Contrast this with calls to methods that are not **static**:

```
Color leaves = new Color( 128, 204, 192 );
int green = leaves.getGreen();
```

The call to getGreen() follows the name of the object (leaves) and not the name of the class (Color). Each Color object carries around its own means of reporting how much green it contains.

7.3 A complete example and an exercise.

Define a class that models lengths that are measured in feet and inches. Use the following class as a model.

```

package weight;

public class Weight {
    public static final int OUNCES.PER.POUND = 16;
    private int pounds;

```

```
private int ounces;

public Weight( int pounds, int ounces ) {
    this.pounds = pounds;
    this.ounces = ounces;
} // Weight( int, int )

public int getPounds() {
    return this.pounds;
} // getPounds()

public int getOunces() {
    return this.ounces;
} // getOunces()

public Weight add( Weight w ) {
    int lbs = this.pounds + w.pounds +
        (this.ounces + w.ounces)/OUNCES.PER_POUND;
    int oz = (this.ounces + w.ounces) % OUNCES.PER_POUND;

    return new Weight( lbs, oz );
} // add( Weight )

@Override
public String toString() {
    return this.pounds + " lbs., " + this.ounces + " oz.";
} // toString()

public static void main( String [] args ) {
    Weight kiwis = new Weight( 1, 8 );
    Weight plums = new Weight( 2, 12 );

    Weight sum = kiwis.add( plums );

    System.out.println( kiwis + " + " + plums + " = " + sum );
} // main( String [] )
} // Weight
```

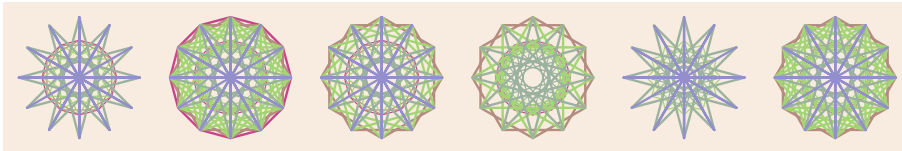
Lesson 8

“Who cares about design unless people want to use it?”

—Linus Torvalds [16, page 80]

“Thus computing is, or at least should be, intimately bound up with both the source of the problem and the use that is going to be made of the answers—it is not a step to be taken in isolation from reality.”

—Richard W. Hamming [11, page 3]



8.1 Video clip

- [IBM Watson: Final Jeopardy! and the Future of Watson](#)

8.2 Exercises

1. In which city in the United States are the largest and second largest airports named, respectively, after a hero of the Second World War and after a great battle of the Second World War?

2. What is the relevance of the previous question about airports to our study of computer science?
3. The world is full of opportunities for curious and energetic people. Most careers are zig-zags, not straight lines. Accidents as much as plans determine most careers. You just never know where you might wind up.

What do Jeff Bates, Matt Harding, Rob Malda, and Eric Weisstein have in common? (Look up the names on the Web!)

4. The Java compiler will reject this code. Fix the code.

```

if( 70 < temp < 75 ) {
    System.out.println( "The room is comfortable." );
} // if

```

5. Computer scientists count from zero. This means that the index of the last element in an array of N items has what value?
6. An array is a homogeneous data structure. A personnel record will probably be a heterogeneous data structure.

What does the word “homogeneous” mean in this context?

7. What is missing in the following code?

```

int [] data;
data[0] = 42;

```

8. Suppose that this next example is an excerpt from a program that computes and reports statistics related to the weather. Suppose that the program in its current form analyzes data for just one year.

A programmer might later adapt the program so that it analyzes the data in a leap year. There are 366 days, not 365 days, in a leap year. Or a programmer might modify the program to work with a measurements collected over decades instead of over a single year.

Replace $i < 365$ in the following code with an expression so that the program will still print the high temperature for every day on which data was recorded even if the programmer changes the size of the data set.

```

double [] temps = new double[365];
for( int i = 0; i < 365; i++ ) {
    System.out.println( "high temperature on day " + i
        " = " + temps[i] );
} // for

```

9. Most **for** loops have a very similar appearance. However, the Java programming language allows many variations.

What is the output of this fragment of a program?

```
for( int i = 0; i < 20; i += 2 ) {  
    System.out.print( i + " " );  
} // for  
System.out.println();
```

10. When they need to sort data, software engineers most often make use of methods that they find in libraries of software. They rarely write their own methods for sorting data. Still, there are good reasons for studying sorting algorithms. What are they?
11. The number of instructions that a computer executes when sorting a list of N items with the selection sort algorithm is proportional to what function of N ?
12. The number of instructions that a computer executes when sorting a list of N items with the insertion sort algorithm is comparable to the number executed when using the selection sort algorithm in the average and worst cases. In the best case, the performance of the insertion sort can be much better than the performance of the selection sort. What is the best case?
13. What is the unit of time that is most relevant to a discussion of events on the microprocessor in your computer?
14. What is the unit of length that is most relevant to a discussion of the transistors and the conducting paths that connect transistors on the microprocessor in your computer?
15. Write code that creates an array of floating point numbers.
16. Write code that assigns a random value in the range 0.0 to 1.0 to each element of an array of floating point numbers.
17. Write code that finds the value of the smallest value in an array of floating point numbers.
18. Write code that finds the position (that is, the index) of the smallest value in an array of floating point numbers.
19. Write code that finds the length of the longest sequence of ascending values in an array of floating point numbers.
20. Write code that prints all pairs of integers (a, b) , where $0 \leq a < N$, $0 \leq b < N$, and $a \neq b$.

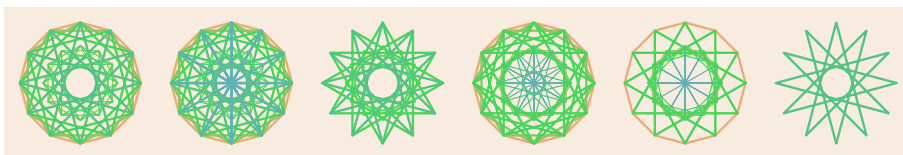
Lesson 9

“Seeing things from a different angle was another strength, and he sometimes made dramatic demonstrations of this. A gymnast in college, Faget liked to leap over chairs in conference rooms or to stand on his head ‘to improve blood circulation in my brain,’ as he put it. With keys and coins falling out of his pockets, Faget would calmly discuss the engineering questions on the agenda.”

—James Oberg [18, page 62]

“Karl Heimburg, chief of the Testing Lab, explained how von Braun went around looking for new ideas, which he would then take to his associates. Heimburg would often listen unimpressed and explain to von Braun why something wouldn’t work. And then Heimburg would find himself saying, ‘But we could do it in this other way,’ and an innovation that he had not considered would have opened up before him.”

—Bruce Murray [17, page 52]



9.1 Exercises

1. Write a method that, given an array of floating point values, returns the sum of those values.
2. Write a method that, given an array of floating point values and a floating point value x , returns an array in which the i^{th} element equals the i^{th} element of the given array divided by x .
3. Write a method that, given an array of floating point values, returns an array in which the i^{th} element equals the sum of the first i elements of the given array.
4. Write a method that, given an array of floating point values and a floating point value x , returns the index of the first element in the array whose value is greater than x . If the array does not contain an element whose value is greater than x , then the array should return the length of the array.

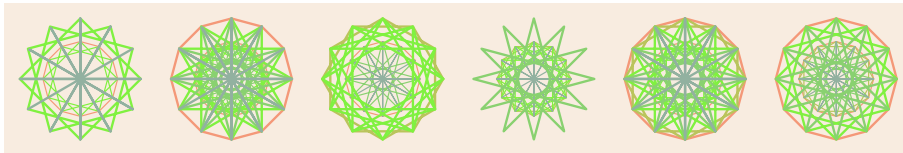
Lesson 10

“Because there is a certain inevitability about inventions of this sort, which have often been anticipated by futurists, the real contribution lies, first, in making the idea work. What is needed next is believing in it passionately enough to supply the emotional energy to carry on the inevitable struggle until the new idea is firmly rooted in the world and has taken on a life of its own. It is a work of intellect and love.”

—Federico Faggin [citesources\[page 102\]](#)gilder

“When I finally quit, I felt weary in my bones. I was actually sweating; my shirt stuck to my back. Things around me kept going in and out of focus. I looked at Alsing, and the rims of his eyes were red. He said he could remember experiencing weariness like this during his midnight-programming days, but he had been younger then. Weariness had been a badge and a part of the fun.”

—Tracy Kidder [13, page 127]



In this lesson, we will learn how to simulate random events on a computer. Random events arise in games. For example, players may draw cards from a shuffled deck or throw dice to determine their next move.

sum	combinations that produce sum					
2	(1,1)					
3	(1,2)	(2,1)				
4	(1,3)	(3,1)	(2,2)			
5	(1,4)	(4,1)	(2,3)	(3,2)		
6	(1,5)	(5,1)	(2,4)	(4,2)	(3,3)	
7	(1,6)	(6,1)	(2,5)	(5,2)	(3,4)	(4,3)
8	(2,6)	(6,2)	(3,5)	(5,3)	(4,4)	
9	(3,6)	(6,3)	(4,5)	(5,4)		
10	(4,6)	(6,4)	(5,5)			
11	(5,6)	(6,5)				
12	(6,6)					

Figure 10.1: Ways to get each possible result in a throw of a pair of dice

A discrete probability distribution function associates a probability with each event in a finite set of possible events. Each probability is a number in the range 0.0 to 1.0. The sum of all probabilities must equal 1.0. This is so because on each trial exactly one of the events must occur. A tossed coin will either land heads up or tails up. The probability of heads plus the probability of tails must equal one—we are certain that one of the two possible outcomes will occur.

Let us think about what can happen when we throw a pair of dice.

Each die has six sides. If the dice are fair, the probabilities of landing with any given side facing up are equal. When the dice land, any one of six numbers could be on the top of the first die and any of six numbers could be on top of the second die—there are 36 possible combinations. Again, if the dice are fair, each of these 36 possible combinations is equally likely.

The sum of numbers on top of the two dice will be in the range 2 to 12. The probability of getting a given sum will depend upon how many combinations can produce the sum. There is only one combination that can produce a sum of 2 (both dice must land with one on top) but there are six combinations that will sum to 7. The probability of rolling a 7 exceeds the probability of rolling a 2 (“snake eyes”).

Figure 10.1 shows the combinations and their sums. In this figure, (i, j) means that the first die lands with i showing on the top face and the second die lands with j showing on its top.

Figure 10.2 shows the probability distribution function. That is, it gives the probabilities of sum with a throw.

The probability that a throw of the dice will yield 2 is about 0.0277. The prob-

sum	# combinations that produce sum	probability of sum
2	1	$\frac{1.0}{36.0} \approx 0.0277$
3	2	$\frac{2.0}{36.0} \approx 0.0555$
4	3	$\frac{3.0}{36.0} \approx 0.0833$
5	4	$\frac{4.0}{36.0} \approx 0.1111$
6	5	$\frac{5.0}{36.0} \approx 0.1388$
7	6	$\frac{6.0}{36.0} \approx 0.1666$
8	5	$\frac{5.0}{36.0} \approx 0.1388$
9	4	$\frac{4.0}{36.0} \approx 0.1111$
10	3	$\frac{3.0}{36.0} \approx 0.0833$
11	2	$\frac{2.0}{36.0} \approx 0.0555$
12	1	$\frac{1.0}{36.0} \approx 0.0277$

Figure 10.2: Probabilities of each possible combination

event	probability
$sum \leq 2$	$\frac{1}{36} \approx 0.0277$
$sum \leq 3$	$\frac{1}{36} + \frac{2}{36} \approx 0.0833$
$sum \leq 4$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} \approx 0.1666$
$sum \leq 5$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} \approx 0.2777$
$sum \leq 6$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} \approx 0.4166$
$sum \leq 7$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} \approx 0.5833$
$sum \leq 8$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} \approx 0.7222$
$sum \leq 9$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} \approx 0.8333$
$sum \leq 10$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} \approx 0.9166$
$sum \leq 11$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} \approx 0.9722$
$sum \leq 12$	$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = 1.0$

Figure 10.3: Cumulative probability distribution for throw of a pair of dice.

ability that a throw will yield 3 is about 0.0555. It follows that the probability of getting 3 or less is $0.0277 + 0.0555 = 0.0833$.

Similarly, the probability that a throw will yield 4 or less is $0.0277 + 0.0555 + 0.0833 = 0.1666$.

Figure 10.3 tabulates the probabilities of outcomes less than or equal to k for all values of k from 2 to 12. It is, of course, certain that a throw of dice will produce an outcome that is less than or equal to one.

10.1 Exercises

These questions refer to the program that follows.

1. The `makeArray()` method returns an array of floating point numbers.
 - (a) How many numbers will the array that `makeArray` returns to its

caller contain?

(b) What is the range of possible values for each element of the array?

```

import java.util.Random;

public class Loops {
    public static final int N = 12;
    public Loops() {
        double [] pdf = makeArray( N );
        normalizeArray( pdf );
        printArray( pdf );
        double [] cdf = accumulate( pdf );
        printArray( cdf );
        int [] histogram = new int[ cdf.length ];
        for( int i = 0; i < 10000; i++ ) {
            int event = randomEvent( cdf );
            histogram[event]++;
        } // for
        for( int i = 0; i < histogram.length; i++ ) {
            System.out.print( histogram[i] + " " );
        } // for
        System.out.println();
    } // Loops()

    private double [] makeArray(int size) {
        double [] result = new double[ size ];
        for( int i = 0; i < result.length; i++ ) {
            result[i] = Math.random();
        } // for
        return result;
    } // makeArray( int )

    private double sumArray( double [] data ) {
        double sum = 0.0;
        for( int i = 0; i < data.length; i++ ) {
            sum += data[i];
        } // for
        return sum;
    } // sumArray( double [] )

    private void normalizeArray( double [] data ) {
        double sum = sumArray( data );
        for( int i = 0; i < data.length; i++ ) {
            data[i] /= sum;
        } // for
    }
}

```

```

} // normalizeArray( double [] )

private double [] accumulate( double [] data ) {
    double [] result = new double[ data.length ];
    double sum = 0.0 ;
    for( int i = 0; i < data.length; i++ ) {
        result[i] = data[i] + sum;
        sum += data[i];
    } // for
    return result;
} // accumulate( double [] )

private int positionOfFirstLargerValue( double [] values ,
    double threshold ) {
    int index = 0;
    while( index < values.length && values[index] < threshold ) {
        index++;
    } // while
    return index;
} // positionOfFirstLargerValue( double [], double )

private int randomEvent( double [] cdf ) {
    double randomNumber = Math.random();
    return positionOfFirstLargerValue( cdf, randomNumber );
} // randomEvent( double [] )

private void printArray( double [] data ) {
    for( double x : data ) {
        System.out.format( "%5.2f ", x );
    } // for
    System.out.println();
} // printArray( double [] )

public static void main( String [] args ) {
    Loops loops = new Loops();
} // main( String [] )
} // Loops

```

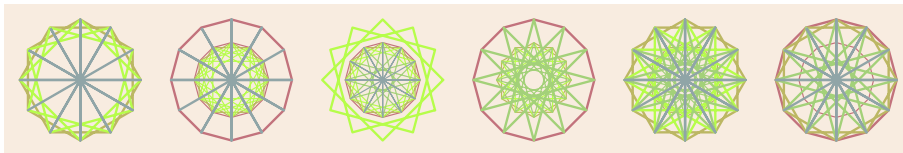
Lesson 11

“No experiment is a failure—even one that doesn’t work. You learn something from them all.”

—Harold E. Edgerton

“Success is the ability to go from failure to failure with no loss of enthusiasm.”

—Winston Churchill



11.1 Video clips

- [IBM Watson: Final Jeopardy! and the Future of Watson](#)
- [Mathworld](#)
- [Slashdot](#)
- [Where the heck is Matt?](#)

1. IBM built and programmed a remarkable computer that beat the best human players at the game of *Jeopardy!* The company has high hopes for the technology it demonstrated in that game. IBM is using the same system now to solve problems that arise in medical care.

The company named the computer after a person who was very important in the company's history. What is the name of the machine?

2. Let's sum the first 8 non-negative integers. We can make the work easier by rearranging and pairing the numbers:

$$\begin{aligned} 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 &= (0 + 7) + (1 + 6) + (2 + 5) + (3 + 4) \\ &= 4 \cdot 7 \\ &= \frac{8}{2} \cdot (8 - 1) \\ &= 28 \end{aligned}$$

What is the sum of the first N non-negative integers?

3. Here is an 8×8 array. The elements on the diagonal are labeled "D." The elements in the lower triangular part of the array (below the diagonal) are labeled "L" and those in the upper triangular array (above the diagonal) are labeled "U."

D	U	U	U	U	U	U	U
L	D	U	U	U	U	U	U
L	L	D	U	U	U	U	U
L	L	L	D	U	U	U	U
L	L	L	L	D	U	U	U
L	L	L	L	L	D	U	U
L	L	L	L	L	L	D	U
L	L	L	L	L	L	L	D

Let's count the number of L's in the array.

Here are two observations that we can use to make the counting easier:

- The array contains $8 \times 8 = 64$ elements. Subtract the 8 diagonal elements and divide what is left by 2 (because what is left contains equal numbers of L's and U's) to get the number of L's.
- The array contains one row that has 0 L's, one row that has 1 L, one row that has 2 L's, and so on to the bottom row, which contains 7 L's.

How many L's are in the array?

4. Here is a trace of a selection sort. The i^{th} row in this table shows the list that is being sorted after the i^{th} step in the execution of the algorithm.

After the i^{th} step, i elements of the list are in order—those elements are printed in boldface type. At each step, the algorithm searches through the unsorted part of the list (the part that is not printed in boldface type) to find the smallest number in that unsorted part. In each search, the algorithm examines all elements in the unsorted part of the list.

How many times does the algorithm examine an element of the list? (Count the numbers in this table that are not printed in boldface type.)

87	32	80	39	15	26	58	70
15	32	80	39	87	26	58	70
15	26	80	39	87	32	58	70
15	26	32	39	87	80	58	70
15	26	32	39	87	80	58	70
15	26	32	39	58	80	87	70
15	26	32	39	58	70	87	80
15	26	32	39	58	70	80	87
15	26	32	39	58	70	80	87

5. Here is a trace of an insertion sort. The i^{th} row in this table shows the list that is being sorted after the i^{th} step in the execution of the algorithm. After the i^{th} step, i elements of the list are in order—those elements are printed in boldface type. At each step, the algorithm searches through the sorted part of the list (the part that is printed in boldface type) to find where it must place the next number from the unsorted part. In the worst case, in each search the algorithm must examine all elements in the sorted part of the list.

How many times does the algorithm examine an element of the list? (Count the numbers in this table that are printed in boldface type.)

42	64	23	43	17	67	19	68
42	64	23	43	17	67	19	68
42	64	23	43	17	67	19	68
23	42	64	43	17	67	19	68
23	42	43	64	17	67	19	68
17	23	42	43	64	67	19	68
17	23	42	43	64	67	19	68
17	19	23	42	43	64	67	68
17	19	23	42	43	64	67	68

6. My favorite definition of computer science answers the question of “What is computer science?” with four more questions:

- What kinds of problems can be solved by writing computer programs?
- Given a solvable problem, how do we write the program?
- Given several programs that solve the same problem, how do we choose the best program?

- Given a claim that a program solves a problem, how can we assure ourselves that it really does solve the problem in all cases?

How does an exercise in counting the number of operations required for sorting a list with the selection sort or insertion sort algorithms relate to this definition?

7. You learned that Eric Weisstein created *www.mathworld.com*, an on-line encyclopedia of mathematics. You learned that Rob Malda and Jeff Bates created *www.slashdot.com* to share news about developments in science and technology. You learned that Matt Harding danced with people in many countries and posted videorecordings of the dancing on *www.wherethehellismatt.com*.

What lesson did you draw from these discoveries?

8. The arithmetic mean of two numbers a and b is $(a + b)/2$. The geometric mean of a and b is $\sqrt{ab} = (ab)^{1/2}$. The harmonic mean of a and b is $2/(1/a + 1/b)$.

It is possible to compute arithmetic, geometric, and harmonic means not only for two numbers, but also for three, four, or many numbers.

Here are methods that compute arithmetic, geometric, and harmonic means. Label each method with the name of the mean it computes and with the value that it returns when $\text{data} = \{2, 4, 8\}$.

```
(a)    public double f1( double [] data ) {
        double product = 1.0;

        for( int i = 0; i < data.length; i++ ) {
            product = product * data[i];
        } // for

        return Math.pow( product , 1.0/data.length );
    } // f1( double [] )
```

```
(b)    public double f0( double [] data ) {
        double sum = 0.0;

        for( int i = 0; i < data.length; i++ ) {
            sum = sum + data[i];
        } // for

        return sum / data.length;
    } // f0( double [] )
```

```
(c)  public double f2( double [] data ) {
        double sum = 0.0;

        for( int i = 0; i < data.length; i++ ) {
            sum = sum + 1.0 / data[i];
        } // for

        return data.length / sum;
    } // f2( double [] )
```

9. Let data = { 17, 11, 19, 41, 43, 13, 31, 29 }.

(a) What value does f(data) return?

(b) What value does g(data) return?

```
public double f( double [] data ) {
    double bestGuessSoFar = data[0];

    for( int i = 0; i < data.length; i++ ) {
        if( data[i] < bestGuessSoFar ) {
            bestGuessSoFar = data[i];
        } // if
    } // for

    return bestGuessSoFar;
} // f( double [] )

public int g( double [] data ) {
    int bestGuessSoFar = 0;

    for( int i = 0; i < data.length; i++ ) {
        if( data[i] < data[bestGuessSoFar] ) {
            bestGuessSoFar = i;
        } // if
    } // for

    return bestGuessSoFar;
} // g( double [] )
```

10. Complete this stub.

```
// Count the number of perfect scores in
// a list of grades.
// Each grade is a number in the inclusive
// range 0 to 100.
// A perfect score is, of course, 100.
```

```

public int countPerfectScores( int [] grades ) {
    return grades.length;
} // countPerfectScores( int [] )

```

11. Suppose that a program includes code that creates a 4×4 array m , initializes it with the values shown, and then calls $f(m)$. The method will change the array. What will the array look like after the call to $f(m)$?

$$m = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{bmatrix}$$

```

public void f( int [][] m ) {
    for( int i = 0; i < m.length; i++ ) {
        for( int j = 0; j < i; j++ ) {
            int temp = m[i][j];
            m[i][j] = m[j][i];
            m[j][i] = temp;
        } // for
    } // for
} // f( int [][] )

```

12. Write code that prints the value of $a^3 + b^3$ for all $0 \leq a < N$ and $0 \leq b < N$.

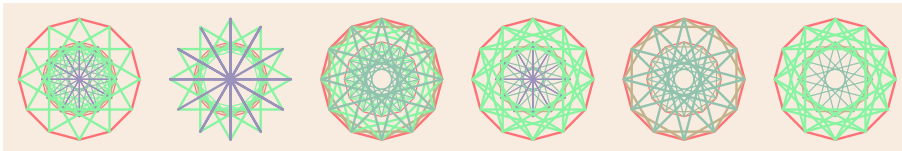
Lesson 12

“You will never remember the many times the launch slipped, but the on-time failures are with you always.”

—Walt Williams [15, page 100]

“Checking doesn’t take place at any one point in a problem-solving situation. Checking permeates the whole procedure.”

—John R. Dixon [6, page 158]



1. Write code that creates an array of integers that is just big enough to hold all of the integers in the arrays a and b.

```
int [] a = {0, 2, 4, 6};  
int [] b = {1, 3, 5, 7};
```

2. Change this code so that it prints the contents of the array on a single line with a space between each pair of consecutive values.

```
int [] c = {0, 1, 2, 3, 4, 5, 6, 7};  
  
for( int i = 0; i < c.length; i++ ) {  
    System.out.println( c[i] );  
} // for
```

3. We can represent a probability distribution function (pdf) with an array. Similarly, we can represent a cumulative distribution function (cdf) with an array.

The values of the elements of the *pdf* are the probabilities of various events. In this example, there are four possible events: Event 0, Event 1, Event 2, and Event 3.

```
double [] pdf = {0.1, 0.2, 0.3, 0.4};
double [] cdf = {0.1, 0.3, 0.6, 1.0};
```

Here is the relationship between the cumulative distribution function and the probability distribution function.

$$\begin{aligned} cdf[0] &= pdf[0] \\ cdf[1] &= pdf[1] + pdf[0] \\ cdf[2] &= pdf[2] + pdf[1] + pdf[0] \\ cdf[3] &= pdf[3] + pdf[2] + pdf[1] + pdf[0] \end{aligned}$$

The rule can be generalized for larger arrays. The value of the i^{th} element of *cdf* is the sum of the values of all elements of *pdf* whose index is less than or equal to i .

$$cdf[i] = \sum_{j=0}^i pdf[j]$$

Complete this stub.

```
// Make a cdf from a pdf.
public double [] makeCDF( double [] pdf ) {
    double [] cdf = new double[ pdf.length ];

    // Add code here.

    return cdf;
} // makeCDF( double [] )
```

4. Let *cdf* represent a cumulative distribution function.

$$cdf = \{0.1, 0.3, 0.6, 1.0\}$$

In this example...

- The probability that the next event will be Event 0 is 0.1.
- The probability that the next event will be either Event 0 or Event 1 is 0.3.
- The probability that the next event will be either Event 0, Event 1, or Event 2 is 0.6.
- The probability that the next event will be either Event 0, Event 1, Event 2, or Event 3 is 1.0. (There are only four possible outcomes in this example—a probability of 1.0 signifies the certainty of choosing one of the events.)

Complete this stub.

```
// Generate a random event with probabilities
// described by a given function.
public int randomEvent( double [] cdf ) {
    int index = 0;
    double r = Math.random();

    // Find the index of the first element
    // of cdf whose value is greater than r.

    return index;
} // randomEvent( double [] )
```

5. It is possible to simulate the behavior of people browsing on the Web with a Markov process. At the heart of the algorithm is the repeated multiplication of a matrix m times a vector v .

$$\begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} m_{00} \cdot v_0 + m_{01} \cdot v_1 + m_{02} \cdot v_2 + m_{03} \cdot v_3 \\ m_{10} \cdot v_0 + m_{11} \cdot v_1 + m_{12} \cdot v_2 + m_{13} \cdot v_3 \\ m_{20} \cdot v_0 + m_{21} \cdot v_1 + m_{22} \cdot v_2 + m_{23} \cdot v_3 \\ m_{30} \cdot v_0 + m_{31} \cdot v_1 + m_{32} \cdot v_2 + m_{33} \cdot v_3 \end{bmatrix}$$

Each element of the product is a sum of products. Each of the products in the sum is obtained by multiplying corresponding elements in a row of m and those in \vec{v} .

Complete this stub.

```
// Compute the sum of the products of corresponding
// elements in two arrays.
public double multiply( double [] row, double [] column ) {
} // multiply( double [], double [] )
```

6. In the following code...

- (a) Circle and label an instance variable.
- (b) Circle and label a local variable.
- (c) Circle and label a constant.
- (d) Circle and label a constructor.
- (e) Circle and label a method that overrides the definition of an inherited method.
- (f) Circle and label an accessor method.
- (g) Circle and label the return type of a method.
- (h) Circle and label the formal parameter of a method.

```

public class Time {
    public static final int MINUTES_IN_AN_HOUR = 60;

    private int hours;
    private int minutes;

    public Time( int hours , int minutes ) {
        this.hours = hours;
        this.minutes = minutes;
    } // Time( int , int )

    public Time add( Time anotherTime ) {
        int h = this.getHours() + anotherTime.getHours();
        int m = this.getMinutes() + anotherTime.getMinutes();
        h = h + m / MINUTES_IN_AN_HOUR;
        m = m % MINUTES_IN_AN_HOUR;
        return new Time( h, m );
    } // add( Time )

    public int getHours() {
        return this.hours;
    } // getHours()

    public int getMinutes() {
        return this.minutes;
    } // getMinutes()

    public String toString() {
        String result = "";
        result += this.getHours() + " hours, ";
        result += this.getMinutes() + " minutes";
        return result;
    } // toString()
} // Time

```


7. Write code that adds 3 hours, 40 minutes to 4 hours, 30 minutes. Use the Time class.
8. Describe what is missing in the following code. It is supposed to merge the contents of a and b but does not finish the job.

It will be sufficient to describe what the missing code must do. You do not have to write the missing code.

```
// This code illustrates the merge operation
// that is at the heart of a merge sort.

// Declare, create, and initialize an array.
// Fill it with an ordered sequence of integers.
int [] a = { 2, 5 , 19, 37, 42, 49, 60, 99 };

// Declare, create, and initialize an array.
// Fill it with an ordered sequence of integers.
int b = { 11, 13, 17, 24, 36, 61, 72, 108 };

// Declare and create an array that
// is just big enough to hold all of the
// elements of a and all of the elements of b.
int [] c = new int[ a.length + b.length ];

// i is an index to an element in a.
int i = 0;
// j is an index to an element in b.
int j = 0;
// k is an index to an element in c.
int k = 0;

// Repeatedly select one element from
// either a or b and copy its value into c.
while( i < a.length && j < b.length ) {
    // Compare the next element in a that
    // has not been copied into c with the
    // next element in b that has not been
    // copied into c.
    if( a[i] < b[j] ) {
        // The element in a is the smaller
        // of the 2 elements—copy it into c.
        c[k] = a[i];
        // Increment the index for a—that is,
        // point to the next element in a.
        i++;
    } // if
```

```

else {
    // The element in b is the smaller
    // of the 2 elements—copy it into c.
    c[k] = b[j];
    // Increment the index for b—that is,
    // point to the next element in b.
    j++;
}
k++;
} // while

// More code needed here...

```

9. How can you tell that this method is recursive?

```

private void flood(int x, int y,
                  int xmin, int ymin, int xmax, int ymax) {

    boolean xInBounds = false;
    if (x > xmin && x < xmax) {
        xInBounds = true;
    } // if

    boolean yInBounds = ymin < y && y < ymax;

    if (xInBounds && yInBounds && !cells[x][y]) {
        cells[x][y] = true;

        flood(x + 1, y, xmin, ymin, xmax, ymax);
        flood(x, y + 1, xmin, ymin, xmax, ymax);
        flood(x - 1, y, xmin, ymin, xmax, ymax);
        flood(x, y - 1, xmin, ymin, xmax, ymax);
    } // if
} // flood( int, int, int, int, int, int )

```

10. Suppose that you have written a collection of Java classes to help other programmers with their projects. How might you distribute your software? (Hint: In what form did the College Board distribute the classes that Advanced Placement students need for exercises with the GridWorld Case Study? What kind of file did you download from our course's Moodle site so that you could complete exercises that Sedgewick and Wayne included in our textbook?)

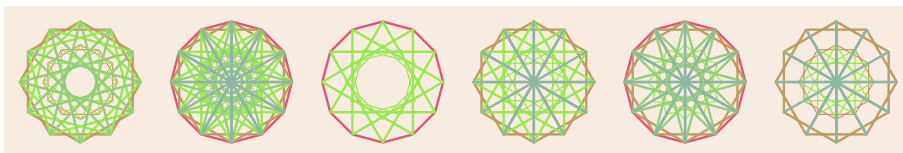
Lesson 13

“Whereas the difference between poor conceptual designs and good ones may lie in the soundness of design method, the difference between good designs and great ones surely does not. Great designs come from great designers. Software construction is a creative process. Sound methodology can empower and liberate the creative mind; it cannot inflame or inspire the drudge.”

—Frederick P. Brooks [3, page 18]

“To make it work they’d have to hire the very best new engineers they could find, ones who would know more about the state of the art in computers than they did. They told each other that they’d have to be sure not to turn away candidates just because the youngsters made them feel old and obsolete; on the contrary, those were the candidates they’d have to welcome.”

—Tracy Kidder [13, page 78]



1. Supply the missing **if** statement to complete the definition of this method.

```
public double findMinimum( double [] data ) {  
    double bestGuessSoFar = data[0];
```

```

    for( int i = 1; i < data.length; i++ ) {
        // Need if statement here.
    } // for

    return bestGuessSoFar;
} // findMinimum( int [] )

```

2. Supply the missing assignment statement to complete the definition of this method.

```

public int findPositionOfMinimum( double [] data ) {
    int bestGuessSoFar = 0;

    for( int i = 1; i < data.length; i++ ) {
        if( data[i] < data[bestGuessSoFar] ) {

            // Need assignment statement here.

        } // if
    } // for

    return bestGuessSoFar;
} // findPositionOfMinimum( double [] )

```

3. Write a method that returns **true** if a given array of integers contains at least one element with a specified value and returns **false** otherwise.

Use this method as a model. You can solve this problem by making small changes to this method.

```

public int countMatches( int [] data, int valueToMatch ) {
    int count = 0;

    for( int i = 0; i < data.length; i++ ) {
        if( data[i] == valueToMatch ) {
            count++;
        } // if
    } // if

    return count;
} // countMatches( int [], int )

```

4. Complete the **for** loop in this method. Be careful! The answer does not begin with **int i = 0**.

```

// Return true if each element in data
// is less than or equal to the element
// that follows.
// Return false otherwise.
public boolean inOrder( int [] data ) {
    boolean result = true;

    for(
        result = result && data[i - 1] <= data[i];
    ) {
        } //

    return result;
} // inOrder( int [] )

```

5. Write a method that computes the weighted average of two floating point numbers. This method will have three parameters: the two numbers to be averaged and the weight. The weight will be a value in the range 0.0 to 1.0. The method will return a floating point value to its caller.

If a and b are the numbers to be averaged and t is the weight, then the weighted average is...

$$\text{weighted - average} = (1 - t) \cdot a + t \cdot b$$

6. I wrote a class that models fractions and then scrambled the order of the lines of code. Reorder these lines to make the class named Rational.

```

line # 0:    } // Rational( int , int )
line # 1: public class Rational {
line # 2:     private int num;
line # 3:         return a;
line # 4:         if( b == 0 ) {
line # 5:             } // if
line # 6:     } // gcd( int , int )
line # 7:     private int den;
line # 8:         return new Rational( n, d );
line # 9:     } // toString()
line #10:         this.den = den/gcd;
line #11: } // Rational
line #12: public Rational( int num, int den ) {
line #13:     public String toString() {
line #14:     public Rational add( Rational r ) {
line #15:         } // else
line #16:         int d = this.den * r.den;
line #17:         this.num = num/gcd;
line #18:     private int gcd( int a, int b ) {

```

```

line #19: } // add( Rational )
line #20: else {
line #21:     int gcd = gcd( num, den );
line #22:     int n = this.num * r.den + this.den * r.num;
line #23:     return gcd( b, a % b );
line #24:     return num + "/" + den;

```

7. This code creates two instances of the Rational classes, computes the sum that the two objects represents, and then prints two identical lines. Explain why the two lines are identical? What is the toString() method?

```

Rational a = new Rational( 1, 2 );
Rational b = new Rational( 1, 6 );
Rational sum = a.add( b );
System.out.println( a.toString() + " + " + b.toString()
    + " = " + sum.toString() );
System.out.println( a + " + " + b + " = " + sum );

```

Write your answer here.

8. We can prove that...

$$\begin{aligned} \gcd(a, b) &= \gcd(b, a \bmod b) \\ \gcd(a, 0) &= a \end{aligned}$$

Here is an example of how Euclid's Algorithm can be used to find the greatest common divisor of two integers.

$$\begin{aligned} \gcd(102, 68) &= \gcd(68, 102 \bmod 68) \\ &= \gcd(68, 34) \\ &= \gcd(34, 68 \bmod 38) \\ &= \gcd(34, 0) \\ &= 0 \end{aligned}$$

This algorithm is recursive.

We know that...

$$a^n = a^{\frac{n}{2}} \cdot a^{\frac{n}{2}} \quad \text{when } n \text{ is even}$$

$$a^n = a \cdot a^{\frac{n}{2}} \cdot a^{\frac{n}{2}} \quad \text{when } n \text{ is odd}$$

$$a^1 = a$$

$$a^0 = 1$$

Here is an example of how to raise a number to a power recursively.

$$2^{16} = 2^8 \cdot 2^8$$

$$2^8 = 2^4 \cdot 2^4$$

$$2^4 = 2^2 \cdot 2^2$$

$$2^2 = 2^1 \cdot 2^1$$

$$2^1 = 2$$

What is gained by using recursion in these two cases?

9. Which of the three sorting algorithms that we examined is recursive?
10. Write a paragraph or two in response to the question: Is computer science a science? Provide an argument in support of your answer.

You might choose to refer to problems you encountered in this course or exercises you completed. Was our work like work that scientists do?

You might find evidence to support your answer in the lives and work of one or more of the computer scientists whom you met through your reading and our discussions. Were their methods or goals like those of scientists?

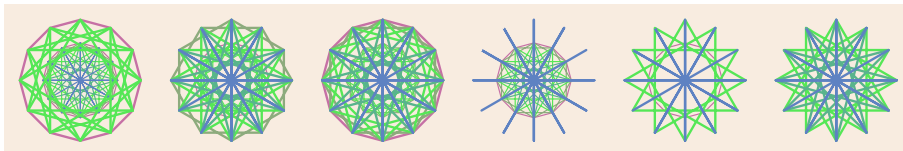
Lesson 14

“It is especially nice when the things we regard as beautiful are also regarded by other people as useful.”

—Donald E. Knuth [14, page 671]

“Beauty is important in engineering terms because software is so complicated. Complexity makes programs hard to build and potentially hard to use; beauty is the ultimate defense against complexity.”

—David Gelernter [8, page 22]



1. My fourth grade teacher asked me and my classmates to write book reports. She asked us to include in our reports definitions of new words that we had encountered in our reading. She forbade us from defining a word using that same word.

Now many years later I am a grown-up computer scientist and know that I can define a method by using the same method. What word names this problem solving technique?

2. Here is a recursive method:

```

public int factorial( int n ) {
    if( n == 0 ) {
        return 1;
    } // if
    else {
        return n * factorial( n - 1 );
    } // else
} // factorial( int )

```

- (a) What is the value of `factorial(4)`?
- (b) For which values of n will the method return $n!$?
- (c) Every recursive method contains an **if** statement (or something equivalent). Why?

3. Here is an iterative method:

```

public int factorial( int n ) {
    int product = 1;
    for( int i = 1; i <= n; i++ ) {
        product *= i;
    } // for
    return product;
} // factorial( int )

```

How does the number of multiplications in an execution of the recursive method compare to the number in an execution of the iterative method?

4. Here are two ways that a teacher of mathematics might present a definition of factorial to students. Which do you prefer? Why?
 - (a) “To compute factorial of n when n is a positive integer, multiply together all of the integers from 1 to n . Let’s also agree that the factorial of 0 will be 1.”
 - (b)

$$\begin{aligned}
 0! &= 1 \\
 1! &= 1 \\
 n! &= n \cdot (n-1)! \quad (\text{for } n > 1)
 \end{aligned}$$

5. Why might you prefer the second of the two versions of `powerOf2()` shown here?

```

// Version #1
public int powerOf2( int exponent ) {
    int power = 1;

    for( int i = 0; i < exponent; i++ ) {

```

```

        power *= 2;
    } // for

    return power;
} // powerOf2( int )

// Version #2
public int powerOf2( int exponent ) {
    if( exponent == 0 ) {
        return 1;
    } // if
    else {
        int product = powerOf2( exponent/2 );
        if( exponent % 2 == 0 ) {
            return product * product;
        } // if
        else {
            return 2 * product * product;
        } // else
    } // else
} // powerOf2( int )

```

6. Refer again to version #2 of powerOf2().

(a) In English, what is the meaning and purpose of the expression `exponent % 2 == 0`?

7. Write a method that computes x^n for any floating point number x and any non-negative integer n ?

8. Write a method that computes x^n for any floating point number x and **any** integer n ?

9. Let a and b be non-negative integers. Let q be the quotient obtained by dividing a by b . Let r be the remainder obtained by dividing a by b .

(a) Write an expression with q , b , and r that completes this equation:

$$a =$$

(b) a and b have a greatest common divisor. Call that divisor d . To say that d is a divisor of a is to say that d divides into a without remainder. Similarly d divides into b without remainder. Let us call the quotients m and n :

$$a = m \cdot d$$

$$b = n \cdot d$$

Rewrite the equation that you wrote for the first part of this question using md in place of a and nd in place of b . Regroup terms to show that if d is a divisor of a and b then it is also a divisor of r .

- (c) Write a method that finds the greatest common divisor of two non-negative integers a and b by testing every possible divisor from 1 up to the smaller a and b .
10. Compute $\text{gcd}(192, 42)$ by using the following definition of the greatest common divisor function and showing the values computed at each step.

$$\begin{aligned}\text{gcd}(a, 0) &= a \\ \text{gcd}(a, b) &= \text{gcd}(b, a \bmod b)\end{aligned}$$

11. The Fibonacci sequence begins: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

After we get past the first two elements, each element is the sum of the two previous elements.

$$\begin{aligned}\text{fib}(0) &= 1 \\ \text{fib}(1) &= 1 \\ \text{fib}(n) &= \text{fib}(n-1) + \text{fib}(n-2) \quad (\text{for } n > 1)\end{aligned}$$

Now let's compute $\text{fib}(5)$.

$$\begin{aligned}\text{fib}(5) &= \text{fib}(4) + \text{fib}(3) \\ &= (\text{fib}(3) + \text{fib}(2)) + (\text{fib}(2) + \text{fib}(1)) \\ &= ((\text{fib}(2) + \text{fib}(1)) + (\text{fib}(1) + \text{fib}(2))) + ((\text{fib}(1) + \text{fib}(0)) + \text{fib}(1))\end{aligned}$$

Imagine computing $\text{fib}(n)$ for some large value of n . What will happen? What's the problem here?

12. We can use Newton's Method to find the roots of equations. In particular, we can find the square root of x by finding the positive root of $x - \text{root}^2 = 0$. (Here x is a given number and root is the unknown value that we are trying to find.)

Newton's Method invites us to guess the answer. Then it gives us a means of using that guess to construct an even better guess. By repeating the process, we can generate successively better approximations of the root we seek.

Let's use a variable named *approx* to hold the value of our current approximation of the square root of a number x .

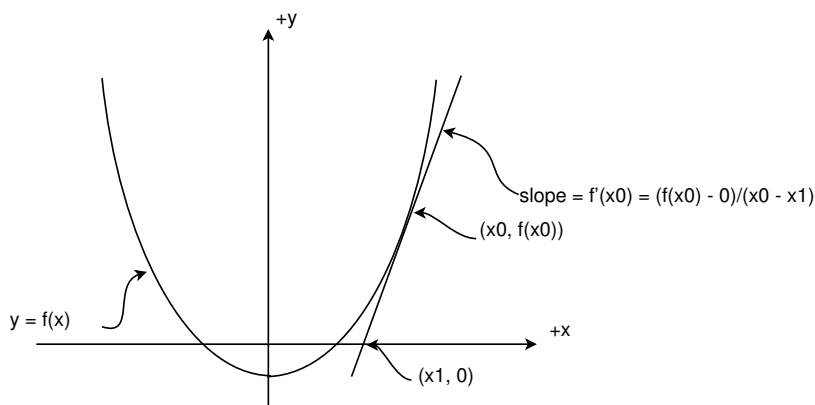


Figure 14.1: Newton's method.

- If *approx* is greater than the square root of x , then x/approx is smaller than the square root.
- If *approx* is less than the square root of x , then x/approx is greater than the square root of x .

It follows that the square root lies between *approx* and x/approx .

Explain what happens in the execution of this method:

```
private static final double EPSILON = 1E-8;

public double squareRoot( double x, double approx ) {
    if( Math.abs( x - approx * approx ) < EPSILON ) {
        return approx;
    } // if
    else {
        double average = (approx + x/approx)/2;
        return squareRoot( x, average );
    } // else
} // squareRoot( double, double, double )
```

13. Here is another derivation of Newton's method. Figure 14.1 shows a plot of a function $f(x)$, a line that is tangent to that curve, the point of tangency, and the point at which the tangent intersects the x axis.

Let us suppose that $f(x) = x^2 - 2$. Then the derivative of $f(x)$ is $f'(x) = 2x$. We will begin by guessing that a root of $f(x)$ is at $x = x_0$. We will produce a better guess and call it x_1 .

Express the x_1 in terms of x_0 , $f(x_0)$, and $f'(x_0)$.

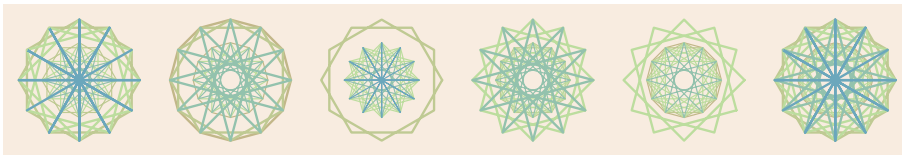
Lesson 15

“I did not distinguish between ‘art’ and ‘science’ and still don’t.”

—Alan Kay [19, page 39]

“The geniuses of the computer field, on the other hand, are the people with the keenest aesthetic senses, the ones who are capable of creating beauty.”

—David Gelernter [8, page 50]



1. Design, write, and test a class that models a vector in two dimensions. Include within your class methods for producing...
 - the sum of 2 vectors: $\vec{u} + \vec{v}$
 - the difference of 2 vectors: $\vec{u} - \vec{v}$
 - the product of a number and a vector: $s\vec{u}$
 - the dot product of two vectors: $\vec{u} \cdot \vec{v}$
 - the magnitude of a vector: $|\vec{u}|$

Also include a method a `toString()` method for producing a string that will give us a printable representation of a vector.

Here's the mathematics that you need to know.

$$\begin{aligned}\vec{u} &= [u_x, u_y] \\ \vec{v} &= [v_x, v_y] \\ \vec{u} + \vec{v} &= [u_x + v_x, u_y + v_y] \\ \vec{u} - \vec{v} &= [u_x - v_x, u_y - v_y] \\ s\vec{u} &= [su_x, su_y] \\ \vec{u} \cdot \vec{v} &= u_x v_x + u_y v_y \\ |\vec{u}| &= \sqrt{\vec{u} \cdot \vec{v}}\end{aligned}$$

Define two constructors.

- A constructor with no parameters will set both elements of a newly created vector equal to zero.
 - A constructor with two parameters will set the elements of a newly created vector equal to the supplied values.
2. Design, write, and test a class that models a 2×2 matrix. Include within your class methods for the producing...

- the sum of 2 matrices: $A + B$
- the difference of 2 matrices: $A - B$
- the product of a number and a matrix: sA
- the product of 2 matrices: AB
- the determinant of a matrix: $|A|$
- the inverse of a matrix (defined only when the determinant is not zero): A^{-1}
- the product of a matrix and a vector: $A\vec{u}$

Also include a method a `toString()` method for producing a string that will give us a printable representation of a matrix.

Here's the mathematics that you need to know.

$$\begin{aligned}
 A &= \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \\
 B &= \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{bmatrix} \\
 A + B &= \begin{bmatrix} (a_{00} + b_{00}) & (a_{01} + b_{01}) \\ (a_{10} + b_{10}) & (a_{11} + b_{11}) \end{bmatrix} \\
 A - B &= \begin{bmatrix} (a_{00} - b_{00}) & (a_{01} - b_{01}) \\ (a_{10} - b_{10}) & (a_{11} - b_{11}) \end{bmatrix} \\
 sA &= \begin{bmatrix} sa_{00} & sa_{01} \\ sa_{10} & sa_{11} \end{bmatrix} \\
 A \times B &= \begin{bmatrix} (a_{00} b_{00} + a_{01} b_{10}) & (a_{00} b_{01} + a_{01} b_{11}) \\ (a_{10} b_{00} + a_{11} b_{10}) & (a_{10} b_{01} + a_{11} b_{11}) \end{bmatrix} \\
 |A| &= a_{00} a_{11} - a_{10} a_{01}
 \end{aligned}$$

$$\begin{aligned}
 A^{-1} &= \frac{1}{|A|} \begin{bmatrix} a_{11} & -a_{01} \\ -a_{10} & a_{00} \end{bmatrix} \\
 A\vec{u} &= \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} \\
 &= \begin{bmatrix} a_{00}u_x + a_{01}u_y \\ a_{10}u_x + a_{11}u_y \end{bmatrix}
 \end{aligned}$$

Define two constructors.

- A constructor with no parameters will set all four elements of a newly created matrix equal to zero.
- A constructor with four parameters will set the elements of a newly created matrix equal to the supplied values.

Finally, it will be convenient to have methods that set the values of the four elements of a matrix in special ways.

$$\begin{aligned}
 \text{makeIdentity}() &\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 \text{makeRotation}(\theta) &\rightarrow \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \\
 \text{makeScaling}(\text{scaleFactor}) &\rightarrow \begin{bmatrix} \text{scaleFactor} & & 0 \\ & 0 & \text{scaleFactor} \end{bmatrix}
 \end{aligned}$$

Vector2D
-x: double -y: double
+Vector2D() +Vector2D(x:double,y:double) +add(otherVector:Vector2D): Vector2D +subtract(otherVector:Vector2D): Vector2D +scale(factor:double): Vector2D +dot(otherVector:Vector2D): double +magnitude(): double +toString(): String

Figure 15.1: Vector2D class.

Matrix2x2
-m: double []
+Matrix2x2() +Matrix2x2(m00:double,m01:double,m10:double,m11:double) +makeIdentity(): void +makeRotation(angle:double): void +makeScaling(factor:double): void +add(otherMatrix:Matrix2x2): Matrix2x2 +subtract(otherMatrix:Matrix2x2): Matrix2x2 +multiply(otherMatrix:Matrix2x2): Matrix2x2 +multiply(factor:double): Matrix2x2 +multiply(u:Vector2D): Vector2D +determinant(): double +inverse(): Matrix2x2 +toString(): String

Figure 15.2: Matrix2x2 class.

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